SCHALLER

Hunting of Synchronous Machines

Electrical Engineering

B. S.

1912
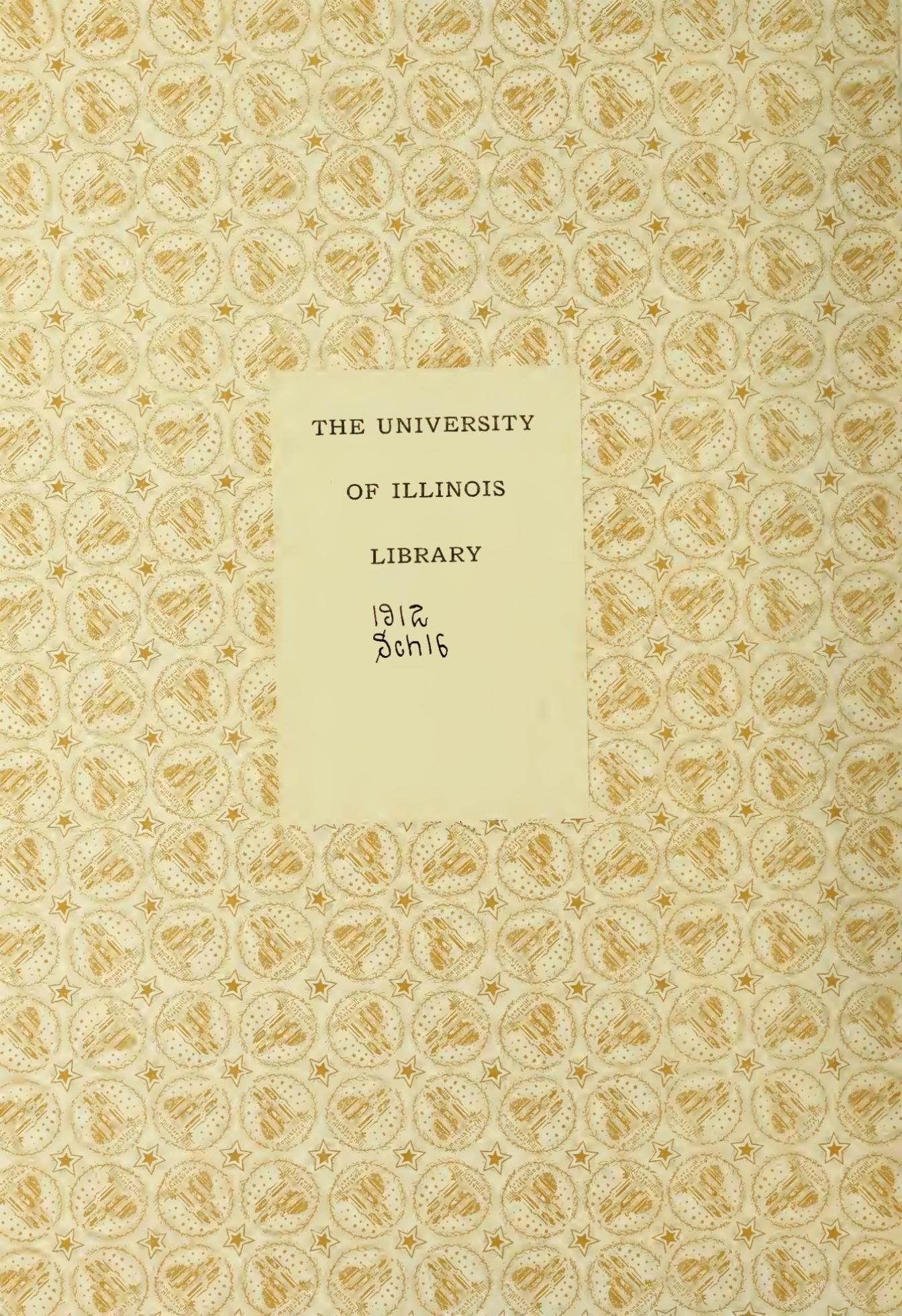
HUNTING OF SYNCHRONOUS MACHINES

BY

WILLIAM FRED SCHALLER

B. S., University of Illinois, 1910

THESIS

Submitted in Partial Fulfillment of the Requirements for the

Degree of

MASTER OF SCIENCE

IN ELECTRICAL ENGINEERING

IN

THE GRADUATE SCHOOL

OF THE

UNIVERSITY OF ILLINOIS

1912

UNIVERSITY OF ILLINOIS

THE GRADUATE SCHOOL

May 31, 1912

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

WILLIAM FRED SCHALLER

ENTITLED HUNTING OF SYNCHRONOUS MACHINES

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

In Charge of Major Work

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Final Examination

HUNTING OF SYNCHRONOUS MACHINES

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HUNTING OF SYNCHRONOUS MACHINES

Introduction and Discussion

The phenomenon of "hunting" has been the chief source of trouble in the operation of synchronous machines. It was first shown by Dr. John A. Hopkins some twenty years ago that when a pair of generators paralleled electrically and running steadily with an equal division of load have their equilibrium of uniform motion disturbed, by, for instance retarding or speeding up one or the other, a balancing force will be set up with a tendency to restore the state of uniform rotation. This force acts to accelerate the slow machine and retard the fast one, thus tending to keep the system in synchronism. This reference applies equally well to synchronous motors or to synchronous condensers fed from alternating current supply mains; there is a tendency to keep the whole system in step.

Any such disturbing force as mentioned results in an oscillation or hunting as the machines come into step. That is, the back machine acquires too much momentum and runs ahead of proper position while the other machine is retarded to a position behind the proper one. These changes of position, in the case of generators, cause unequal division of load. When a machine is ahead it takes more than its share and is retarded; when it is behind it takes less than its share and the synchronizing force furnishes the tendency

to speed up and gather momentum. Such an oscillation brings
about a transfer of power in a flow of cross current between
the machines which depends for its value on the amplitude of
the oscillation. The result is that the interchange of
power creates an alternate motor-generator action by each
machine. The oscillation is finally damped out by the action
of the retarding forces, friction, windage and the losses due
to the flow of the cross currents. If conditions in the
machine are such that all damping effect is counter-acted
the oscillations continue indefinitely at constant amplitude.
If damping effect is negative the hunting will be cumulative
and the machines will fall out of step.

After recognition of the existance of such a damp-
ing effect the study of the subject led to investigations of
fly wheel action and effect of the rotating parts and finally
to the development of the amortisseur windings in pole faces.

The four chief sources of damping effect are:

1. Friction
2. Air resistance
3. Local armature currents
4. Variation of field of machine caused by
the oscillation.

The first and third of these increase approximately in pro-
portion to the velocity; the second in proportion to the
square of the velocity. Variations of the field produce
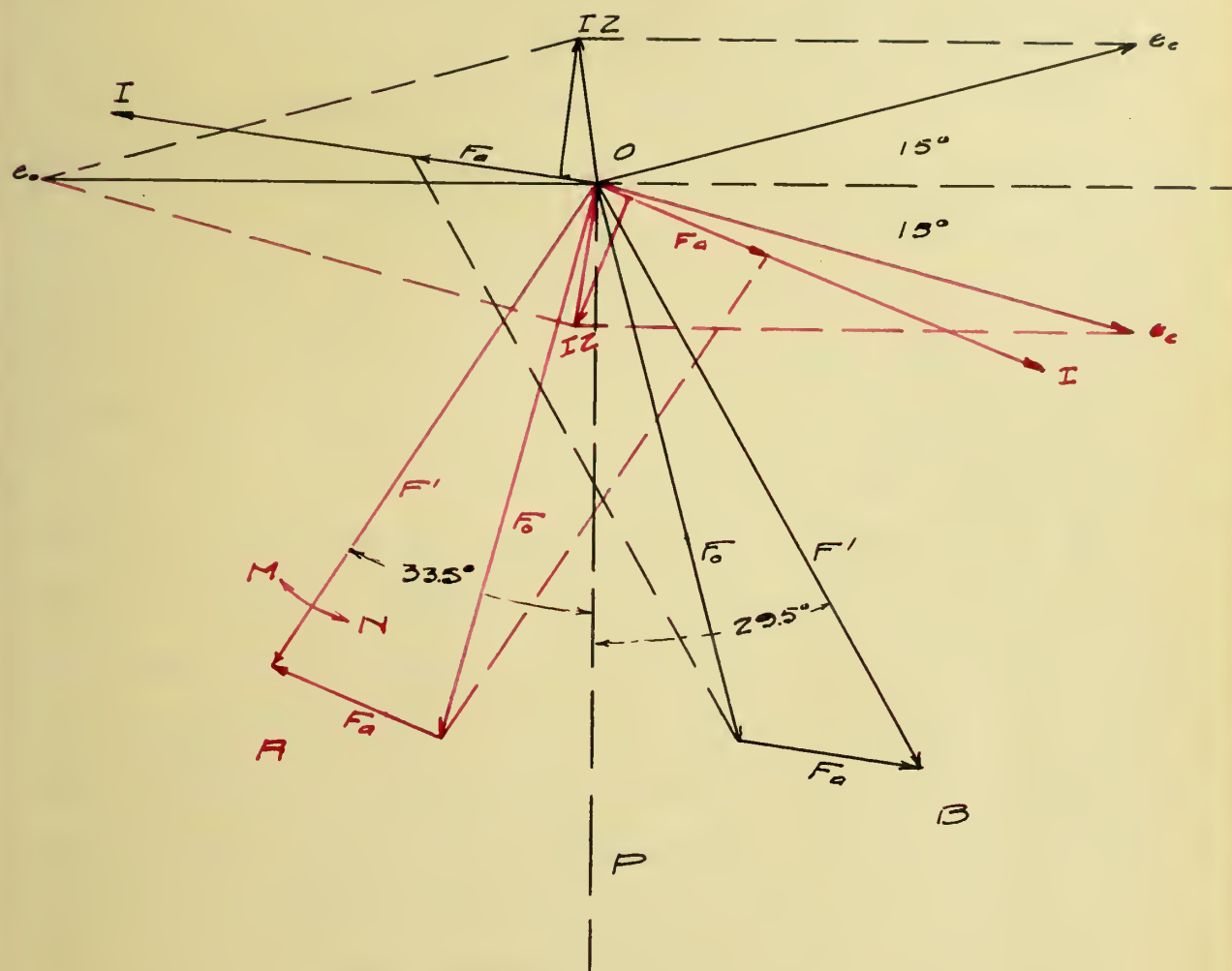eddies in the iron of the poles and these with the changes of

Figure 1

e_o Impressed E.M.F.-Gen.   Arm. React 30%

e_c Counter E.M.F. Motor   No Transmission Loss

$X = 4r$

field current in reacting on the armature cause torque changes
which have the desired damping effect.

Fig. 1 shows a vector analysis of the phenomenon
of hunting. A vector displacement of counter E. M. F. of
motor of 15° on each side of direct opposite to impressed
E. M.F. is assumed. No transmission loss is allowed for so
that IZ drop is the cross E. M. F. between the machines. The
current direction is found from the reactance-resistance re-
lation $x = 4r$. F_0, the excitation causing the counter E.M.F.,
is at right angles with e_c and has the same value in both
positions. The excitation which must be furnished by the
field is F, and is found by combination of the armature re-
action of 30% with F_0. Hunting may be represented by two
kinds of motion, namely, free and forced oscillations. In
any case, to have hunting, the motion, as an oscillation, must
be harmonic in nature. Since hunting is a mechanical phenom-
enon it must be represented by a shift of vector positions of
flux values. This means then that the hunting action is re-
presented by the oscillation of the vector F' across some
line, as a pendulum string vibrates across a perpendicular.
The question of distinction between free and forced oscilla-
tion now comes up. Reference to the vector diagram of the
alternator, Fig. 2, which need not be further explained, (P. 6)
shows that the direction of F' is deflected from the verti-
cal as load comes on. In order then, to have hunting with
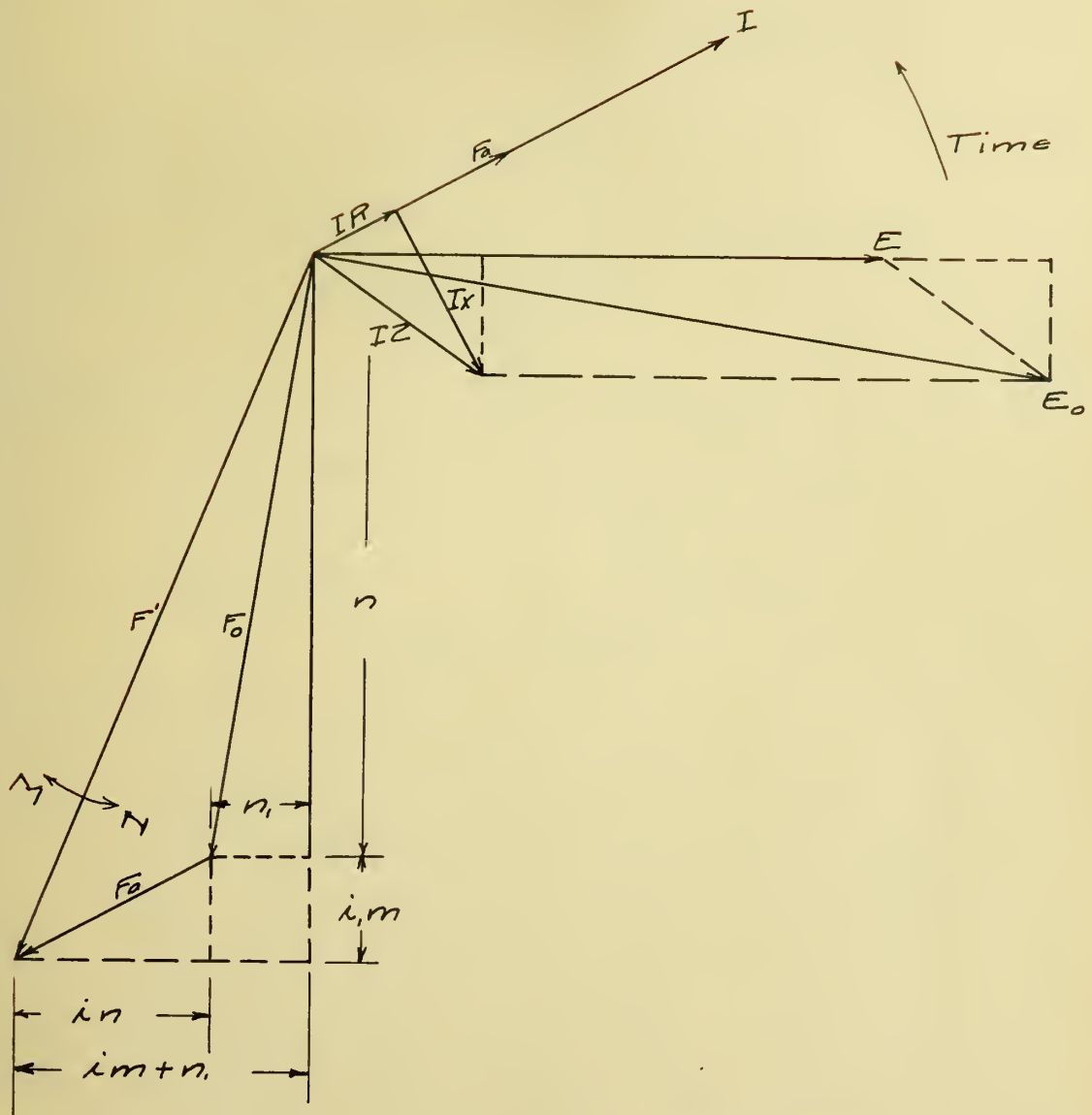load on the machine, the vector F' must oscillate across its

Figure 2
Vector Diagram
of.
Alternator
Lagging Current

Explanation -- Fig. 2 -- Page 5.

$$E_0 = E + I Z = E + (i + j i_1) (r - j x)$$

$$= (E + ir + i_1 x) - j (i x - i_1 r)$$

$$\quad\quad\quad n \quad\quad\quad\quad\quad\quad\quad n_1$$

Get n and n_1 from saturation curve

$$F_0 = - j n - n_1 = (i + j i_1) m + F'$$

$$m = \frac{armature\ reaction}{current}$$

For round rotor, uniform magnetic reluctance

Single phase $\quad\quad\quad m = \frac{1}{2} \sqrt{2}\ T\ (\sin \alpha\ )$

Two phase $\quad\quad\quad\quad m = \sqrt{2}\ \ T$

Three phase $\quad\quad\quad m = 1.5\ \sqrt{2}\ \ T$

T = number of turns

For definite pole correct thus

m_1 = armature reaction for wattless I

m = armature reaction for energy I = $\frac{2}{3}\ m_1$

x_1 = self inductive reaction for wattless I

x = self inductive reaction for energy I = 1.5 x_1

* normal position over the arc M N, Fig. 1. This represents a
forced oscillation, and is caused by some external force as
a pulsation of prime mover speed, a sudden change of load or
a momentary short circuit. A free oscillation means that load
is thrown off, consequently the vibration of F' will take place
about the vertical O P over the arc A B as indicated. In this
paper consideration will be taken of free oscillations only.

Further discussion of Fig. 1 brings up the following two points. In the diagram a shift of equal amplitude of counter E. M. F. was used, with the result that the flux angles on the two sides of O P are not of the same value. Since they must be the same, according to our definition of hunting it follows that the counter E. M. F., e_c, will have to swing thru unequal angles on each side of direct opposite to the impressed E. M. F. The value of F' varies considerably in its swing as indicated in positions of maximum displacement in the diagram. It is seen that there must be a corresponding change in field current value. This is a logical conclusion because there is a change of field current in the case of alternator short circuit, and hunting is a specific case of short circuit. An oscillogram of the field current of a hunting machine shows a pulsation at hunting frequency.

II   Mathematical Treatment.

The hunting of a machine, as a simple harmonic motion, will have the same motion equation as a spring provided with a damping dash pot, and represented diagrammatically in Fig. 3.

The equation may be set up thus:

$$L = J \frac{d^2 \theta}{d t^2} + K \frac{d \theta}{d t} + a \theta$$

where

$J = M \rho^2$ = Moment of inertia

L = disturbing moment

K = moment of retarding force per unit angular velocity.

a = twisting moment per unit angular displacement

This is a forced oscillation. Now remove L and a free oscillation takes place, with the equation:

$$J \frac{d^2\theta}{dt^2} + K \frac{d\theta}{dt} + a\theta = 0.$$

The solution of this equation is the equation of motion of a hunting machine when proper values are used for J, K & a.
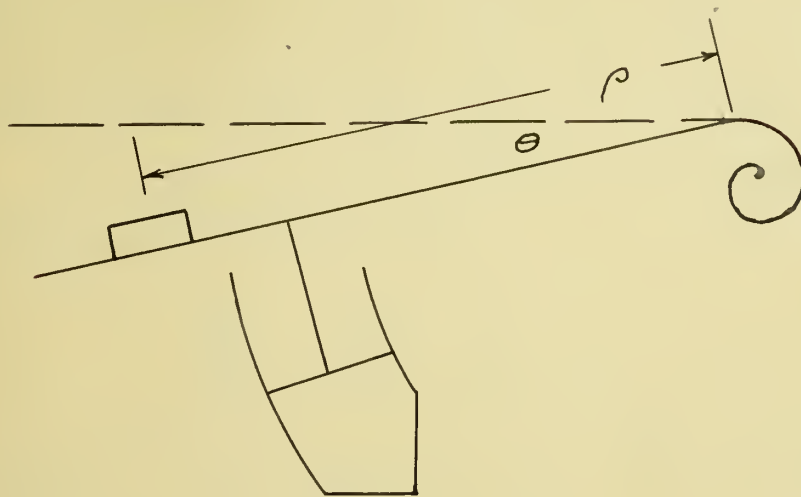

Figure 3

Each of the three terms represents a moment inside the armature, and their sum must equal the moment of disturbing force, zero in this case.

J is the moment of inertia of the rotating part, and

$J \dfrac{d^2 \theta}{dt}$ represents the moment required for its acceleration.

a is the twisting moment per unit angular displacement; it is the torque necessary to give unit displacement of the rotating parts.

K is the moment of retarding force per unit angular velocity furnished by the eddies in the pole faces or losses in the amortisseur winding. $K \dfrac{d\theta}{dt}$ has a value in the equation depending on the angular velocity of the hunting vibration, and represents any loss which varies in that manner.

The equation of hunting is of the second order and first degree, and has the form

$$\frac{d^2 y}{dx^2} + A \frac{dy}{dx} + B y = C$$

The solution if this, when A, B, and C are constants and not functions of x is, by Euler's method:

$$y = A_1 \varepsilon^{m_1 x} + A_2 \varepsilon^{m_2 x} + \frac{C}{B}.$$

Since, then

$$\frac{d^2 \theta}{dt^2} + \frac{K}{J} \frac{d\theta}{dt} + \frac{a}{J} \theta = 0$$

$$\theta = A_1 \varepsilon^{m_1 t} + A_2 \varepsilon^{m_2 t}$$

Where m_1 and m_2 are roots of the equation

$$m^2 + \frac{K}{J} m + \frac{a}{J} = 0$$

Solving

$$m_1 = -\frac{K}{2\ J} + \sqrt{\frac{K^2}{4\ J^2} - \frac{a}{J}}$$

$$m_2 = -\frac{K}{2\ J} - \sqrt{\frac{K^2}{4\ J^2} - \frac{a}{J}}$$

So that

$$\theta = A_1\ \varepsilon^{\left(-\frac{K}{2\ J} + \sqrt{\frac{K^2}{4\ J^2} - \frac{a}{J}}\right)\ t} + A_2\ \varepsilon^{\left(-\frac{K}{2\ J} - \sqrt{\frac{K^2}{4\ J^2} - \frac{a}{J}}\right) t}$$

or

$$\theta = \varepsilon^{-\frac{K}{2\ J}\ t}\left[A_1\ \varepsilon^{\ t\ \left(\sqrt{\frac{K^2}{4\ J^2} - \frac{a}{J}}\right)} + A_2\ \varepsilon^{-\ t\ \sqrt{\frac{K^2}{4\ J^2} - \frac{a}{J}}}\right]$$

Since ordinarily K is small the square root becomes negative, and we may write

$$\theta = \varepsilon^{-\frac{K}{a\ J}\ t}\left[A_1\ \varepsilon^{+\ j\ t\ \sqrt{\frac{a}{J} - \frac{K^2}{4\ J^2}}} + A_2\ \varepsilon^{-\ j\ t\ \sqrt{\frac{a}{J} - \frac{K^2}{4\ J^2}}}\right]$$

By expansion into series and comparison

$$\theta = \varepsilon^{-\frac{K}{2\ J}\ t}\left[A_3\ \sin\left(t\sqrt{\frac{a}{J} - \frac{K^2}{4\ J^2}} + B_3\right)\right]$$

or

$$\theta = A_3\ \varepsilon^{-\frac{K}{2\ J}\ t}\left[\sin\left(t\sqrt{\frac{a}{J} - \frac{K^2}{4\ J^2}} + B_3\right)\right]$$

Now, when $t = 0$, $\theta = \theta_0$,

where θ_0 is maximum displacement in oscillation.

then

$$\theta_0 = A_3\ \sin B_3$$

$$A_3 = \frac{\theta_0}{\sin B_3}$$

Differentiate the equation containing A_3 and B_3:

$$\frac{d\theta}{dt} = -A_3 \frac{K}{2J} \varepsilon^{-\frac{K}{2J}t} \sin\left(t\sqrt{\frac{a}{J} - \frac{K^2}{4J^2}} + B_3\right)$$

$$+ A_3 \sqrt{\frac{a}{J} - \frac{K^2}{4J^2}} \; \varepsilon^{-\frac{K}{2J}t} \cos\left(t\sqrt{\frac{a}{J} - \frac{K^2}{4J^2}} + B_3\right)$$

From the previous condition, when $t = 0$, the angular velocity $\dfrac{d\theta}{dt} = 0$.

Then

$$\sqrt{\frac{a}{J} - \frac{K^2}{4J^2}} \; \cos B_3 = \frac{K}{2J} \sin B_3$$

$$\tan B_3 = \frac{2J\sqrt{\frac{a}{J} - \frac{K^2}{4J^2}}}{K}$$

and $\displaystyle B_3 = \tan^{-1}\frac{2J\sqrt{\frac{a}{J} - \frac{K^2}{4J^2}}}{K}$

also $\displaystyle A_3 = \frac{\theta_0}{\sin\left[\tan^{-1}\dfrac{2J\sqrt{\frac{a}{J} - \frac{K^2}{4J^2}}}{K}\right]}$

Substituting

$$\theta = \frac{\theta_0}{\sin\left[\tan^{-1}\dfrac{2J}{K}\sqrt{\frac{a}{J} - \frac{K^2}{4J^2}}\right]} \; \varepsilon^{-\frac{K}{2J}t}$$

$$\sin\left[t\sqrt{\frac{a}{J} - \frac{K^2}{4J^2}} + \tan^{-1}\frac{2J}{K}\sqrt{\frac{a}{J} - \frac{K^2}{4J^2}}\right]$$

As a further reduction it may easily be shown from Fig. 4 that

$$\tan^{-1} x = \sin^{-1} \frac{x}{\sqrt{1 + x^2}}$$
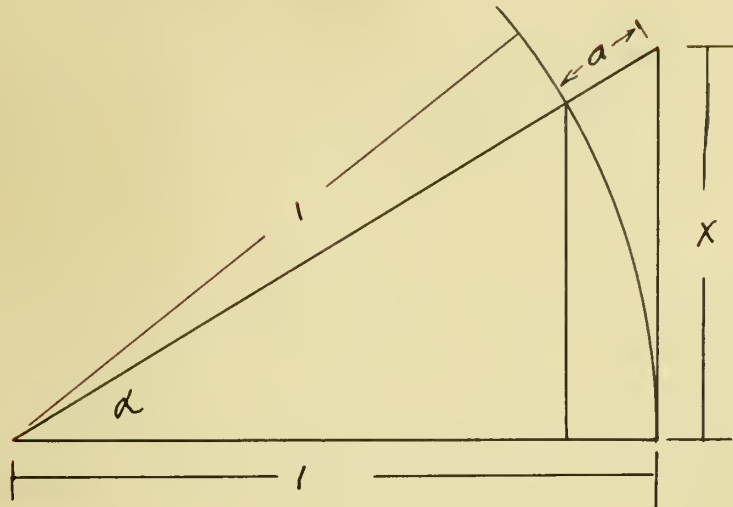
Figure 4

since $\sin (\sin^{-1} a) = a$

we get by substituting

$$\theta = \frac{\dfrac{\theta_o}{\dfrac{2J}{K}\sqrt{\dfrac{a}{J} - \dfrac{K^2}{4 J^2}}}}{\sqrt{1 + \left(\dfrac{2J}{K}\sqrt{\dfrac{a}{J} - \dfrac{K^2}{4 J^2}}\right)^2}} \varepsilon^{-\frac{K}{2J} t}$$

$$\sin\left[t\sqrt{\dfrac{a}{J} - \dfrac{K^2}{4 J^2}} + \sin^{-1} \dfrac{\dfrac{2J}{K}\sqrt{\dfrac{a}{J} - \dfrac{K^2}{4 J^2}}}{\sqrt{1 + \left[\dfrac{2J}{K}\sqrt{\dfrac{a}{J} - \dfrac{K^2}{4 J^2}}\right]^2}}\right]$$

On simplification

$$\Theta = \frac{\Theta_0}{\sqrt{1 - \frac{K^2}{4\,J\,a}}}\;\varepsilon^{-\frac{K}{2\,J}\,t}$$

$$\sin\left[\,t\sqrt{\frac{a}{J} - \frac{K^2}{4\,J^2}} \;+\; \sin^{-1}\sqrt{1 - \frac{K^2}{4\,J\,a}}\,\right]$$

The equation as given holds only for free oscilla-
tions. If the oscillations are forced, as, for example, when
in the case of an engine driven alternator the power supplied
varies periodically from a constant value due to speed fluc-
tuation in the prime mover, the general equation for the
motion may be set up as follows:

$$\frac{d^2\Theta}{d\,t^2} \;+\; \frac{K}{J}\,\frac{d\,\Theta}{d\,t} \;+\; \frac{a}{J}\,\Theta = L\,\sin\omega\,t.$$

The solution is

$$\Theta = \frac{L}{\sqrt{\frac{K^2}{J^2}\,\omega^2 + \left(\frac{A}{J} - \omega^2\right)^2}}\;\sin\left(\omega\,t + \beta\right)$$

where β is the phase angle.

Coming back now to the equation for free oscilla-
tion it is in order to determine the hunting frequency. For
one complete vibration the angle Θ varies from 0 to 2π.
Therefore, the time of one complete period is

$$T = \frac{2\pi}{\sqrt{\dfrac{a}{J} - \dfrac{1}{4}\dfrac{K^2}{J^2}}}$$

From this the beats per second are

$$f_H = \frac{1}{T} = \frac{\sqrt{\dfrac{a}{J} - \dfrac{K^2}{4\,J^2}}}{2\pi}$$

Now get an expression for a. In any machine

$$\frac{K.\ W.}{.746} = \frac{R.\ P.\ M.\ \cdot\ 2\pi\rho'\cdot F}{33000}$$

where

ρ' is radius of the rotor in feet.

F is force in pounds.

Then torque

$$T = F\rho' = \frac{7050\cdot K.\ W.}{R.\ P.\ M.}$$

By definition, if θ_1 is angular displacement in space in radians

$$a = \frac{T}{\theta_1} = \frac{7050\ K.\ W.}{R.\ P.\ M.\ \cdot\ \theta_1}$$

Expressing θ_1 in electrical units, from

$$\theta_1 = \frac{\theta\cdot 2}{p}$$

where p is the number of poles,

we have

$$a = \frac{7050\cdot K.\ W.\ \cdot\ p}{2\,\theta\cdot R.\ P.\ M.}$$

Since frequency per second is

$$f = \frac{p}{2} \cdot \frac{R.\,P.\,M.}{60}$$

we may write

$$a = \frac{7050 \cdot K.\,W. \cdot 60\ f}{\theta \cdot R.\,P.\,M.^2} = \frac{423000\ K.\,W. \cdot f}{\theta \cdot R.\,P.\,M.^2}$$

so that

$$f_H = \frac{\sqrt{\dfrac{a}{J} - \dfrac{K^2}{4\ J^2}}}{2\ \pi} = \frac{\sqrt{\dfrac{423000 \cdot K. \cdot W. \cdot f}{J. \ \theta. \ R.\,P.\,M.^2} - \dfrac{K^2}{4\ J^2}}}{2\ \pi}$$

$$= .159 \sqrt{\frac{423000\ K.\,W. \cdot f}{J \cdot \theta \cdot R.P.M.^2} - \frac{K^2}{4\ J^2}}$$

When the last term is neglected due to the low
value of K the expression becomes

$$f_H = \frac{103.6}{R.P.M.} \sqrt{\frac{K.W. \cdot f}{J\ \theta}}$$

with θ in radians. When θ is expressed in electrical
degrees

$$f_H = \frac{103.6\ \sqrt{57.3}}{R\ P\ M} \sqrt{\frac{K.\,W. \cdot f}{J\ \theta}}$$

or

$$f_H = \frac{785}{R\ P\ M} \sqrt{Q\ \frac{f}{\theta}}$$

where $Q = \dfrac{K.\,W.}{J}$.

It now remains to deduce a general expression for
the constant K. As previously stated, K depends for its value
on any loss which varies with the velocity of the vibrating
mass. Since due to the general low frequency of hunting the
variation of air resistance met during the vibration is
negligible, resulting in a loss perhaps of 1 watt in 100 K.W.,
the only appreciable losses of a K nature are those due to
eddy currents set up in the pole faces and to losses in the
amortisseur or, short circuited rotor, winding. Most field
poles on modern machines are laminated so that, with low fre-
quency, the loss due to eddies in the iron is kept down to a
very low value. The loss in the rotor winding is by far the
greatest component of K, therefore it alone will be consider-
ed. Let us investigate the action of the amortisseur winding.

The amortisseur winding consists of bars of some
conducting material, as copper or brass, which are imbedded
in the pole face. The question of their pitch arrangement
will be discussed later. These bars are short circuited by
heavy end rings, thus giving practically an induction motor
rotor. The motion of hunting must be considered as a devia-
tion, slower and faster, from absolutely uniform rotation at
synchronous speed. For purposes of loss calculation, then,
the motion may be considered as an oscillation about any
one fixed position. In Fig. 5 the oscillation takes place
between - θ and + θ. Assume a north pole. Then, as move-
ment takes place toward the right or + θ there is a relative
movement of flux toward the left and cutting the conductors,

so that weak and strong pole tips are developed. The crowd-
ing of flux toward the trailing tip T means that the conductors
at that end are cut by fewer lines of flux than those near
the leading tip, with the result that a lesser E. M. F. is
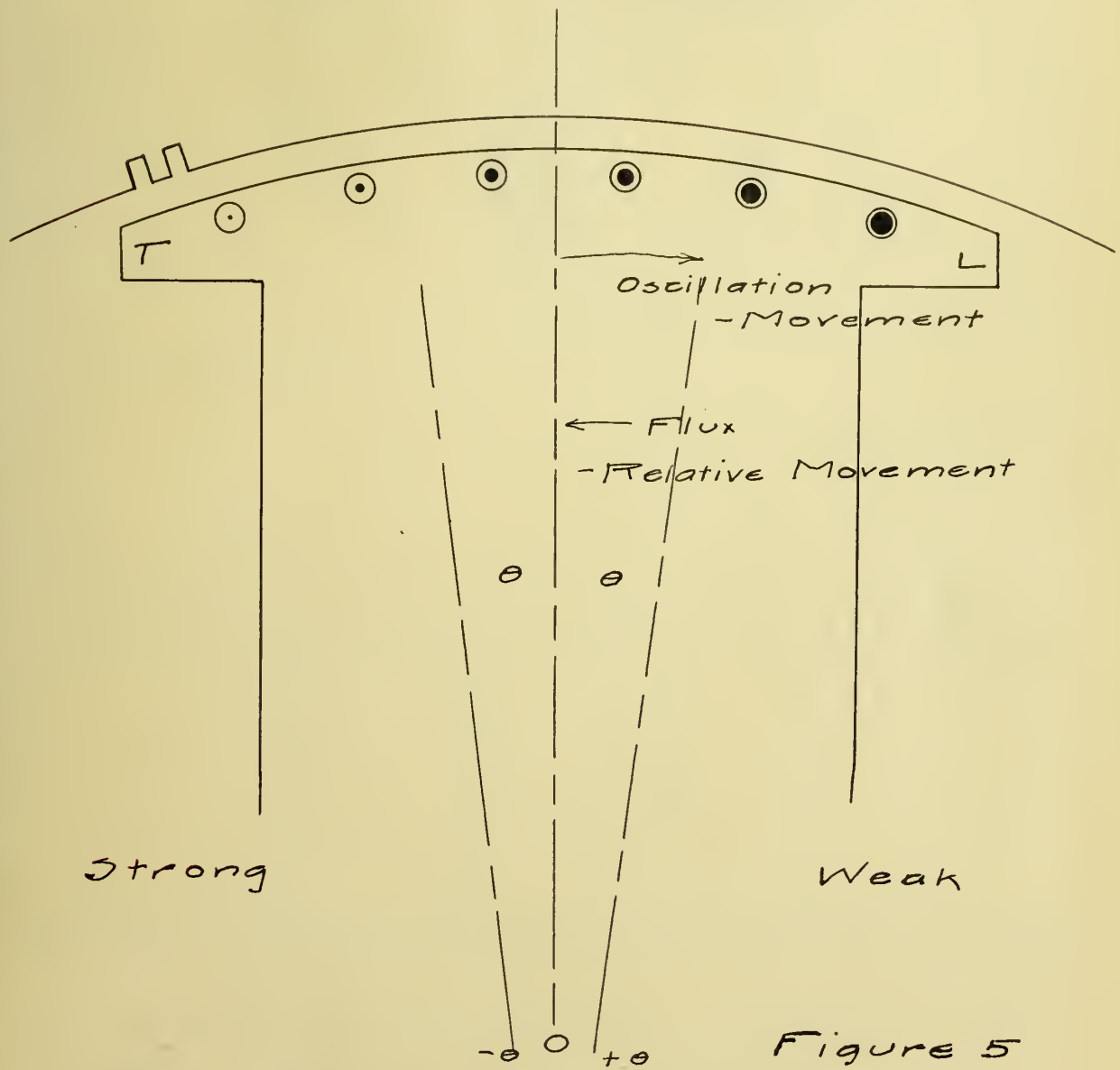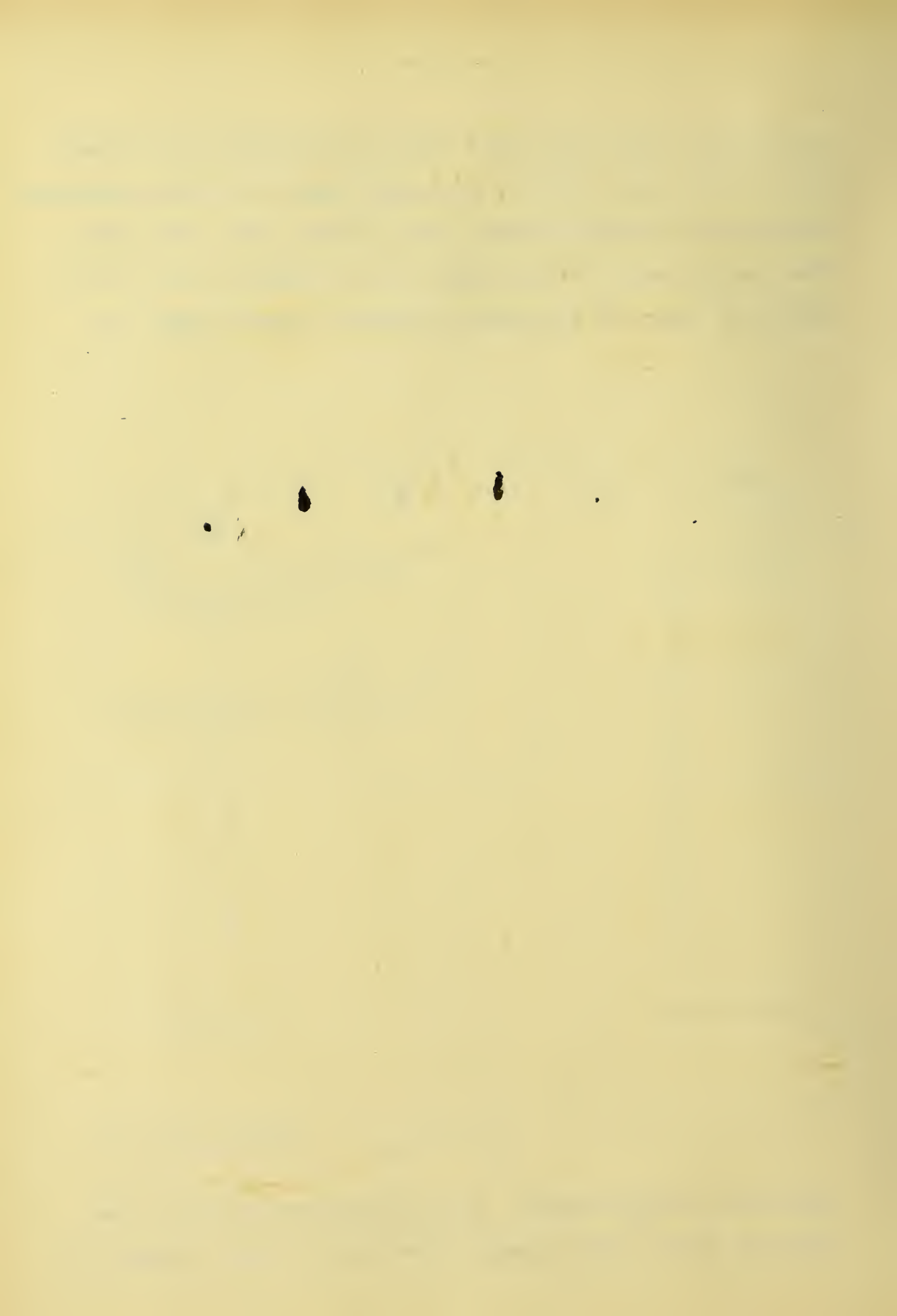set up in them and therefore a smaller current flows, as

Figure 5

indicated on the diagram. In the next pole, however, the
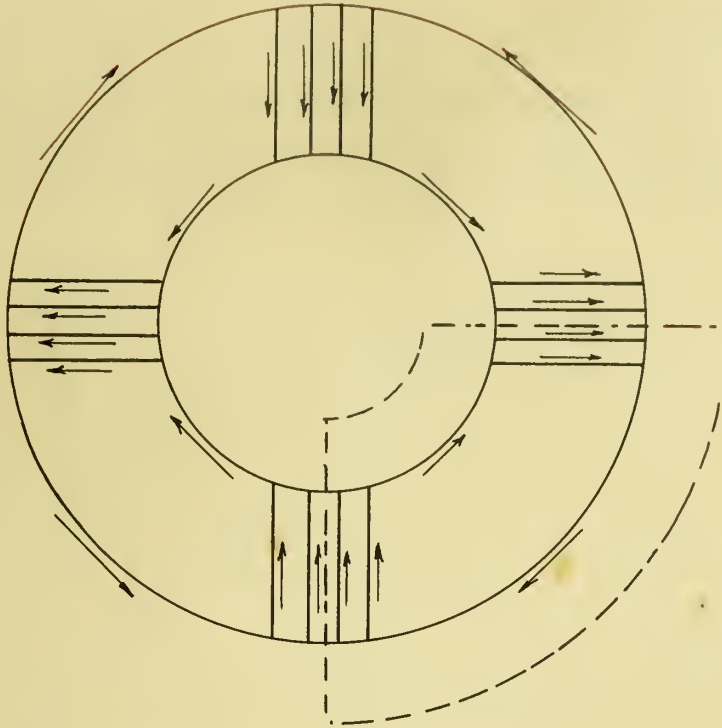currents flow in the opposite direction so that a system of
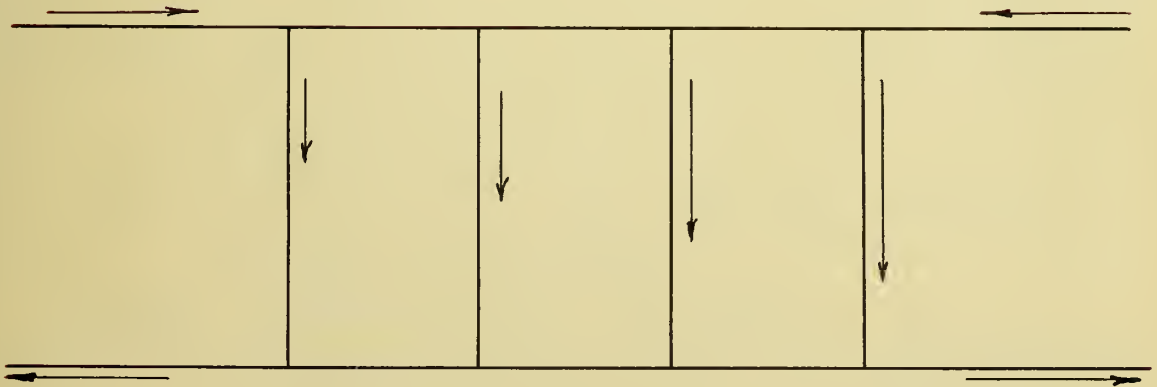
Figure 6

Figure 7

circuits as indicated in Figs. 6 and 7 is developed. The currents which flow in the bars are somewhat out of phase with each other as indicated in Fig. 8, but this effect is
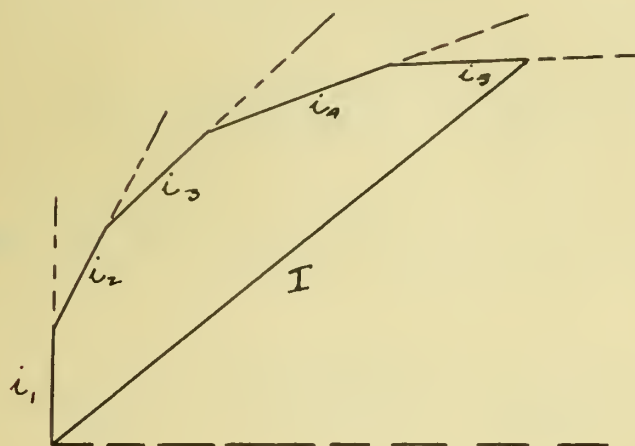
Figure 8

very small and negligible. Due to this, however, and to the fact that the currents in the bars are not of the same value the current paths are not as simple as laid out in Fig. 6.

Let us derive an expression for K

Let B = flux density, lines per square centimeter.

ρ' = radius of amortisseur

winding = radius of rotor, centimeters.

l = effective length of rotor bars, centimeters

R = rotor winding resistance per circuit, ohms.

The instantaneous E. M. F. induced per bar is

$$e = l \, B \, \rho' \frac{d\,\theta}{d\,t} \; 10^{-8}$$

where $\rho' \dfrac{d\theta}{dt}$ is velocity. Since two such E. M. F.s in the same direction are acting in each circuit the value 2 e must be used. The instantaneous power per circuit is

$$P' = 2\,e\,i = \frac{4\,e^2}{R}$$

$$P' = \frac{4\,l^2\,B^2\,\rho'^2}{R} \left(\frac{d\theta}{dt}\right)^2 10^{-16}$$

There are as many circuits as poles, therefore total power is

$$P = p\,P' = \frac{4\,p\,l^2\,B^2\,\rho'^2}{R} \left(\frac{d\theta}{dt}\right)^2 \cdot 10^{-16}$$

Now,   (Torque = moment)

       Power = Torque x Velocity

       Instantaneous torque

$$= K\,\frac{d\theta}{dt} = F\rho'$$

where F is a force pulling on the rotor due to damping.

Then damping power is

$$F\,\rho'\,\frac{d\theta}{dt} = \frac{4\,p\,l^2\,B^2\,\rho'^2}{R} \left(\frac{d\theta}{dt}\right)^2 10^{-16}$$

By a transfer into common units

$$F\,\frac{\rho'}{30.4} \cdot \frac{d\theta}{dt} \cdot \frac{746}{550} = \frac{4\,p\,l^2\,B^2\,\rho'^2}{R} \left(\frac{d\theta}{dt}\right)^2 10^{-16}$$

or

$$F \frac{\rho'}{30.4} \frac{d\theta}{dt} = \frac{2.95 \; p \; l^2 \; B^2 \rho'^2}{R} \left(\frac{d\theta}{dt} \right)^2 10^{-16}$$

Where both sides are in ft. lbs.

$\frac{d\theta}{dt}$ is in radians

B in lines per sq. cm.

ρ' in cm.

l in cm.

F in lbs.

R in ohms.

Since, however,

$$F \frac{\rho'}{30.4} = K \frac{d\theta}{dt}$$

we may substitute

$$K \frac{d\theta}{dt} \cdot \frac{d\theta}{dt} = \frac{2.95 \; p \; l^2 \; B^2 \rho'^2}{R} \left(\frac{d\theta}{dt} \right)^2 10^{-16}$$

or

$$K = \frac{2.95 \; p \; l^2 \; B^2 \rho'^2}{R} \; 10^{-16}$$

NOTE:--Page 22.
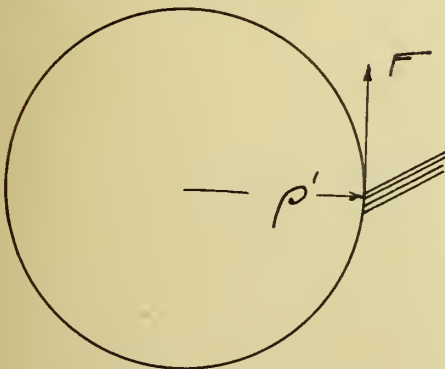
Figure 9

The retarding force acting to prevent hunting and caused by the damping acts as a friction at radius ρ', of the amortisseur winding, from the center of the shaft. The familiar power equation is

$$HP = \frac{2\pi\rho' \cdot R\,P\,M \cdot F}{33000}$$

NOTE:--In order to form a simple engineering conception of K let us develop a new constant K_1 which is analogous to K in its nature. It is to represent the retarding effect of a friction acting on the surface of the rotor, under the consideration that the armature rotates at hunting frequency. The value of K as just derived is the one to be used in the equation for hunting. It will be found that K_1 is a multiple of K and that its relation to losses is exactly the same.

$$H.P. = \frac{2 \pi \rho' \cdot R.P.M. \cdot F}{33000}.$$

This means that the power expended thru an oscillation is translated into a power expended thru a complete rotation of 360 degrees, taking the time of one complete period, as is shown more fully for the E. M. F. calculation on pages 28, 29 & 30. The R. P. M. value then is the number of beats per minute. We may write then, using K_1 for K.

$$K_1 \frac{d\theta}{dt} = \rho' \cdot F = \frac{HP. \cdot 33000}{2\pi \cdot R.P.M.} = \frac{HP. \cdot 33000}{2\pi \cdot 60 \cdot f_H}$$

Substituting

$$K_1 \frac{d\theta}{dt} = \frac{33000}{2\pi \cdot 60 \cdot f_H} \cdot \frac{2.95 \, p \, 1^2 \, B^2 \rho'^2}{33000 \, R} \left(\frac{d\theta}{dt} \right)^2 10^{-16}$$

But

$$\frac{d\theta}{dt} = \frac{2 \cdot \theta_0 \cdot f_H}{57.3} \cdot 60 \quad \text{rad. per min.}$$

$$K_1 = \frac{33000}{2\pi \cdot 60 \cdot f_H} \cdot \frac{2.95 \, p \, 1^2 \, B^2 \rho'^2}{33000 \, R} \cdot \frac{2 \, \theta_0 \, f_H}{57.3}, \; 60 \cdot 10^{-16}$$

Since, however, $\frac{2\theta_0}{57.3}$ has been considered as one complete revolution, we have

$$K_1 = \frac{33000}{2\pi \cdot 60 \cdot f_H} \cdot \frac{2.95 \, p \, 1^2 \, B^2 \rho'^2}{33000 \, R} \cdot 60 \, f_H \cdot 10^{-16}$$

$$= \frac{2.95}{2\pi} \; \frac{p \, 1^2 \, B^2 \rho'^2}{R} \; 10^{-16}$$

$$= .47 \; \frac{p \; l^2 \; B^2}{R} \rho'^2 \; 10^{-16}$$

This value of K_1 is _not_   to be used in the original equation.

III Numerical Example

It is practically impossible to calculate exact values for a machine because of the inexact knowledge of many of the conditions. The determination of rotor resistance when the number of bars per pole exceeds four is particularly cumbersome. A calculation for the very simplest case of amortisseur application follows. The following approximations, as made, illustrate what the difficulties in the way of an exact calculation would be.

1.   The E. M. F. per rotor bar is calculated. Taking as resistance, r, a value 1.2 r', where r' is the actual resistance of one bar, the current is found, and from this the loss per bar is calculated. This value when multiplied by the total number of bars in the winding gives the total loss in the amortisseur. The 1.2 coefficient allows twenty per cent for end ring resistance. It must be remembered that there is a large drop due to the flowing of the high currents over the contact resistance at the points where the rotor bars are joined to the end rings. Then as the temper-

ature of the winding increases its resistance changes materially.

2.   Assume an oscillation thru the maximum angle of hunting.

3.   Assume a condition of no damping to get the frequency of hunting by the expressions derived above.

4.   Neglect eddy losses in pole faces and rotor winding.

5.   Assume a uniform flux distribution. This means that the crowding of flux toward either pole tip is neglected: that each rotor bar cuts flux of the same density thruout its travel, so that the same E. M. F. is induced in each and so the same loss per bar is obtained. This assumption with the previous one neglects circulating currents in the pole faces due to eddies, phase difference of currents in individual bars and unequal values of those currents. An attempt will be made later to calculate the effect of eddy currents.

6.   It is permissible to neglect the slot reactance in calculating the hunting loss in the amortisseur on account of the low frequency.

The conditions described above will be used in the calculation of losses in an amortisseur winding of various resistances placed in the rotor of a turbo-alternator having the following constants and carrying full load at unity power factor.

1.   A T B 12 - 300 - 600 - 2300 Volts.

2.   Round rotor, therefore uniform magnetic reluctance.

3.   Weight of revolving element - 1500 lbs.

4.   Radius of gyration 1.3 feet.

5.   Radius of rotor 1.5 feet.

6.   Moment of inertia

$$J = M \rho^2 = \frac{1500}{32.2} \cdot 1.3^2 = 78.7 \text{ lb. ft.}^2$$

7.   Air gap - .32 inch.

8.   Breadth of pole face 10 in.

9.   Slots per pole per phase - 2.

10.   Average gap density (2300 volts) - 60000 lines per square inch.

11.   Synchronous reactance A. T. - 2400.

12.   Armature reaction A. T. - 1910.

13.   Armature resistance - 2%.

14.   By use of relations

$$\text{Arm. React.} = 1.5 \sqrt{2} \ I \ t$$

and

$$E = 4.44 \ \Phi \ t \ f \cdot 10^{-8}$$

the total number of lines per pole is found to be $\underline{4.16 \cdot 10^6}$

15.   With a density of 60000 lines per square inch the pole face area is

$$\frac{4.16 \cdot 10^6}{60000} = 70 \text{ sq. in.}$$

16.   With a breadth of 10 inches the length of pole face arc is $\frac{70}{10}$ or 7 inches, approximately 73% of the pole
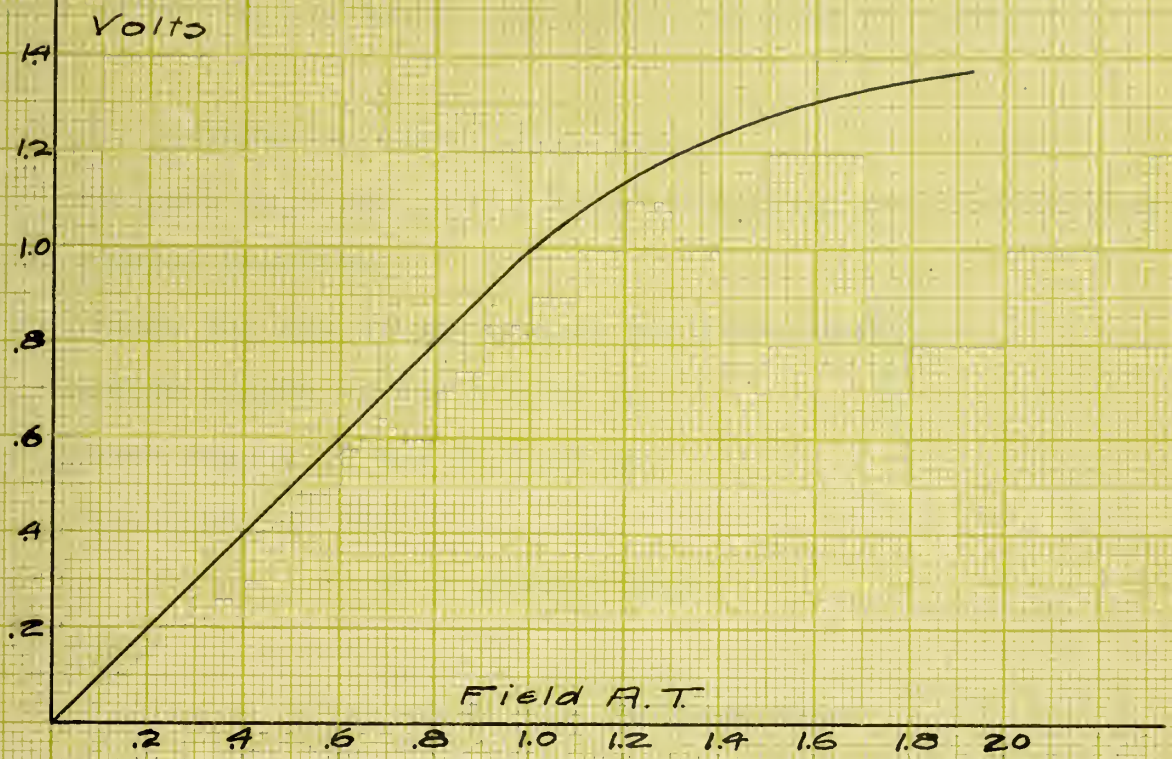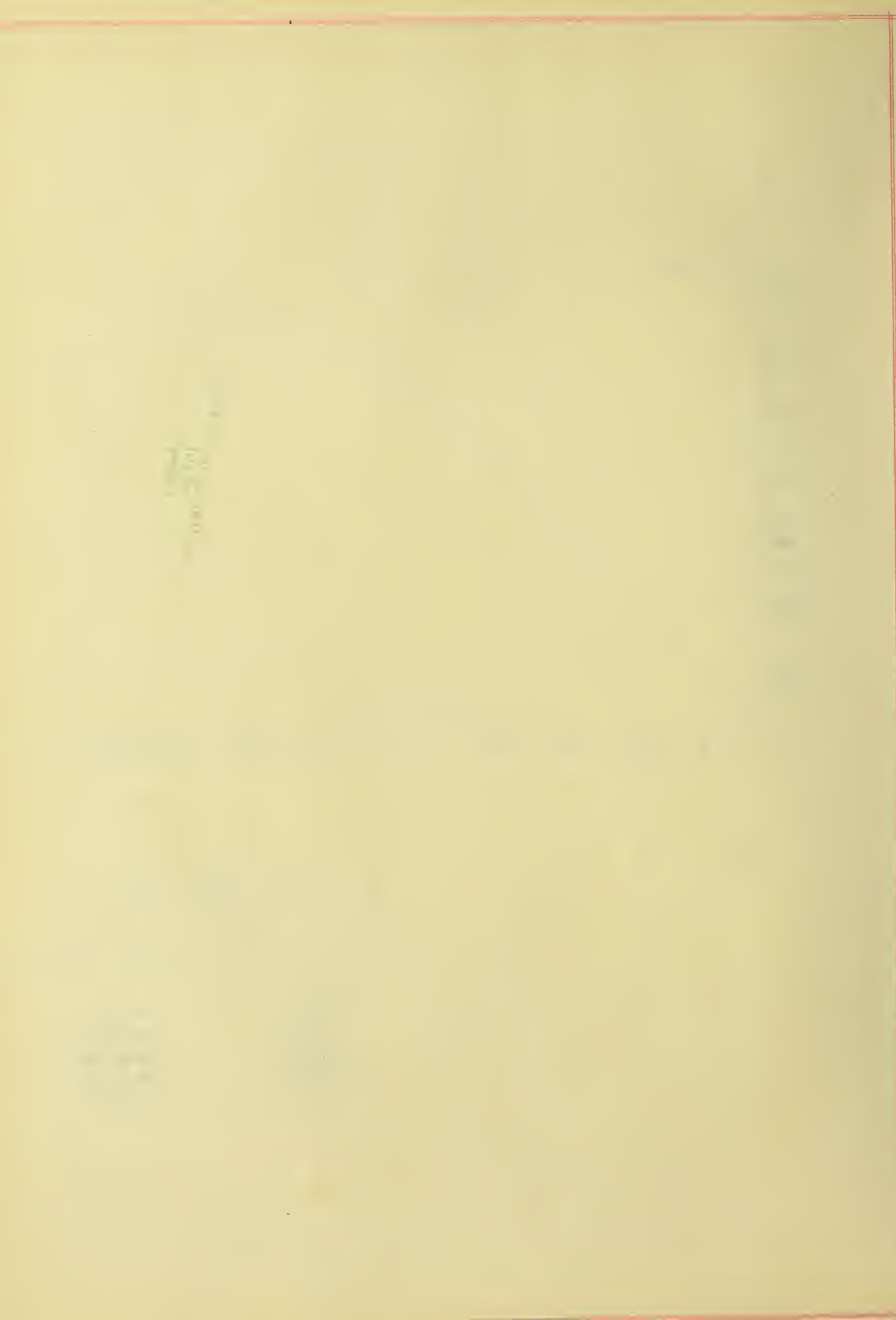
Figure 10
Saturation Curve
for
ATB-12-300-600-2300V.

Data

Volts	AT
1150	3230
2300	6600
2870	9300
3020	10680

arc.

17. Assume width of slot equal to width of tooth.
Then, since there are two slots per pole per phase the slot
pitch works out 1.6 in. so that width of slot or tooth is
.8 in.

18. Saturation curve, Fig. 10.

Assume 6 bars per pole. This assumption is made to agree
with general practice.

Assume the use of #0000 wire for rotor bars.

Active length of bar l = 10 in.

Total length of bar l' = 12 in.

Resistance of bar

$$r' = .000049 \text{ ohms.}$$

Then

$$r = 1.2 \, r' = .0000588 \text{ ohms.}$$

In order to find the effective value of induced
E. M. F. per bar it is found convenient to consider any
single complete vibration or oscillation as a complete rota-
tion thru 360 electrical degrees. Consider, as in Fig. 11
a case of oscillation of a pole piece thru 2 θ space degrees.
Count time beginning at A. Since the flux tends to remain
stationary, or rather to rotate synchronously, the greatest
number of lines will be cut in the shift thru the first $\Delta \theta$
from uniform rotation, then fewer and fewer, till at the end
of the arc of oscillation, in the time Δt while reversal is
taking place, no lines are cut. The result is shown in the

A

20

Figure 11

Lines
Cut

Reversal

Time

Start

Back
at
Start
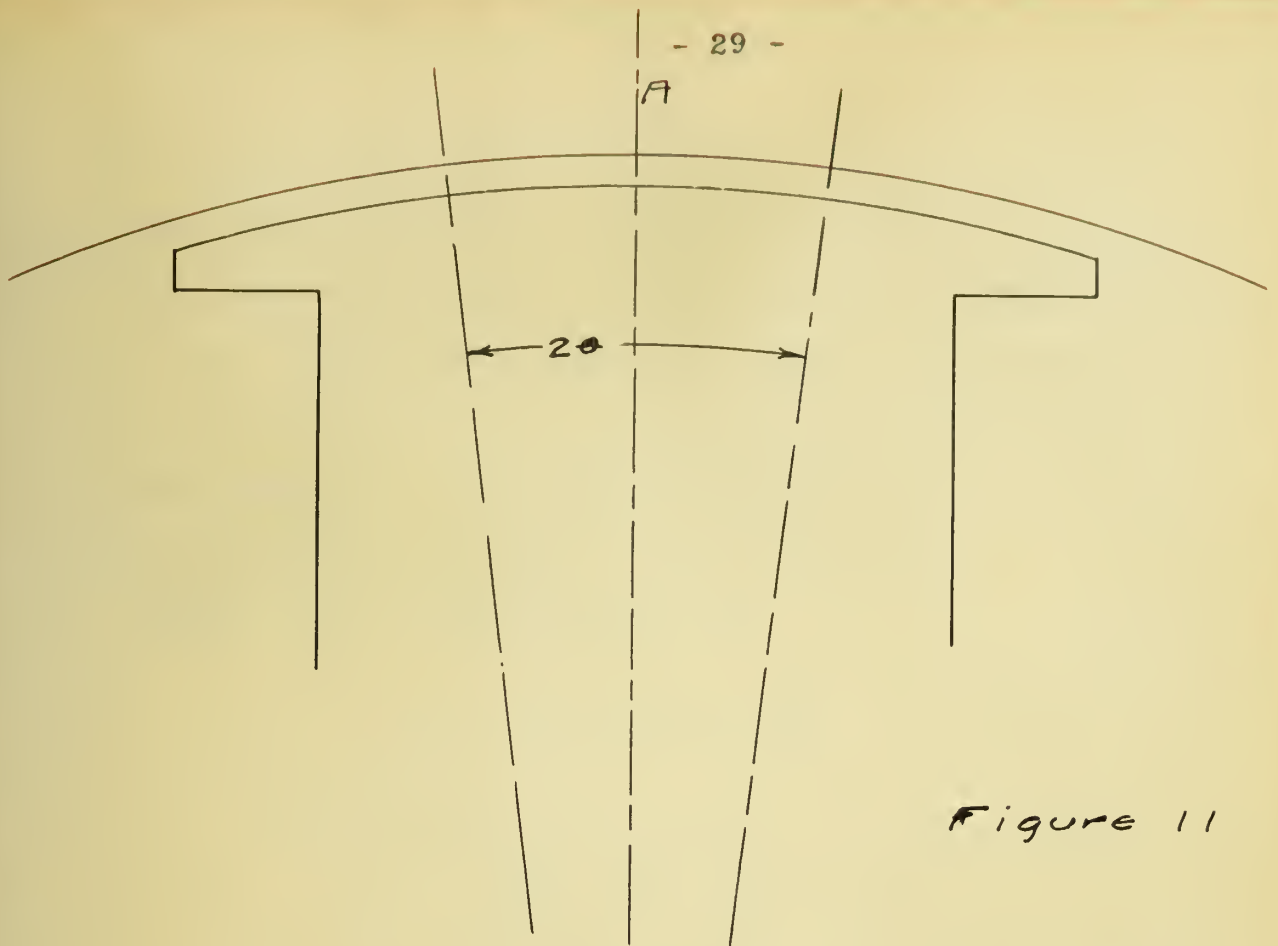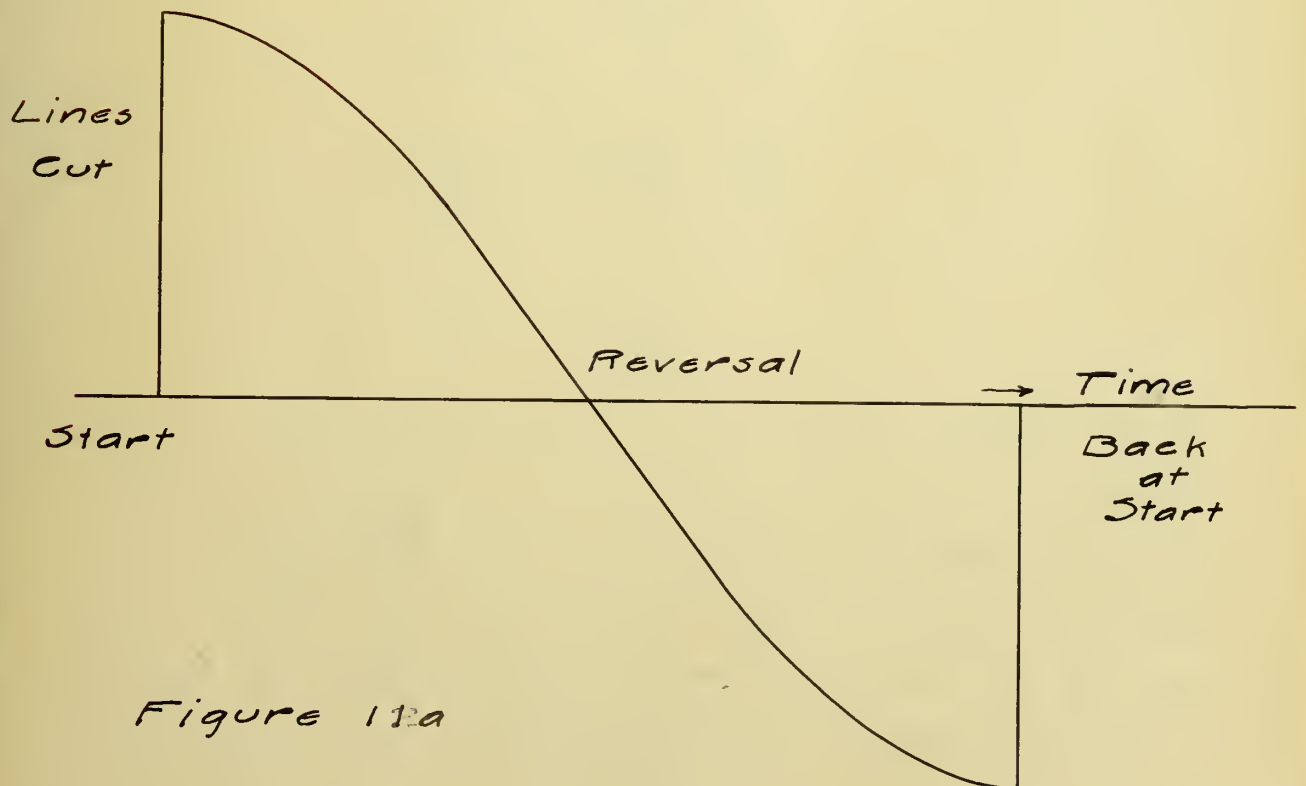
Figure 11a

curve in Fig. 11a. The curve represents a cosine function because the hunting of an electrical machine is a harmonic motion. By a continuation of the motion it is apparent that its result represents the voltage induced in each rotor bar of a generator with an amortisseur for a winding and run at hunting frequency. The general voltage equation for a concentrated winding may then be used, as follows:

The E. M. F. induced per bar is

$$E = 4.44 \ f_H \ N \ \Phi \cdot 10^{-8}$$

As previously found

$$f_H = \frac{785}{R \ P \ M} \sqrt{Q \cdot \frac{f}{\Theta^\circ}}$$

$$= \frac{785}{R \ P \ M} \sqrt{\frac{K. \ W.}{J} \cdot \frac{f}{\Theta^\circ}}$$

$$= \frac{785}{600} \sqrt{\frac{300}{78.8} \cdot \frac{60}{\Theta^\circ}}$$

$$= 19.8 \cdot \frac{1}{\sqrt{\Theta^\circ}}$$

The value of Θ° may be calculated from the vector diagram of the machine as follows. The assumption has been made that the machine is carrying full load current at unity power factor. The vector F will then be displaced as shown by angle Θ_0 which, when the machine is released by the load being thrown off, is the maximum angle of hunting. This value is to be used in calculating f_H.
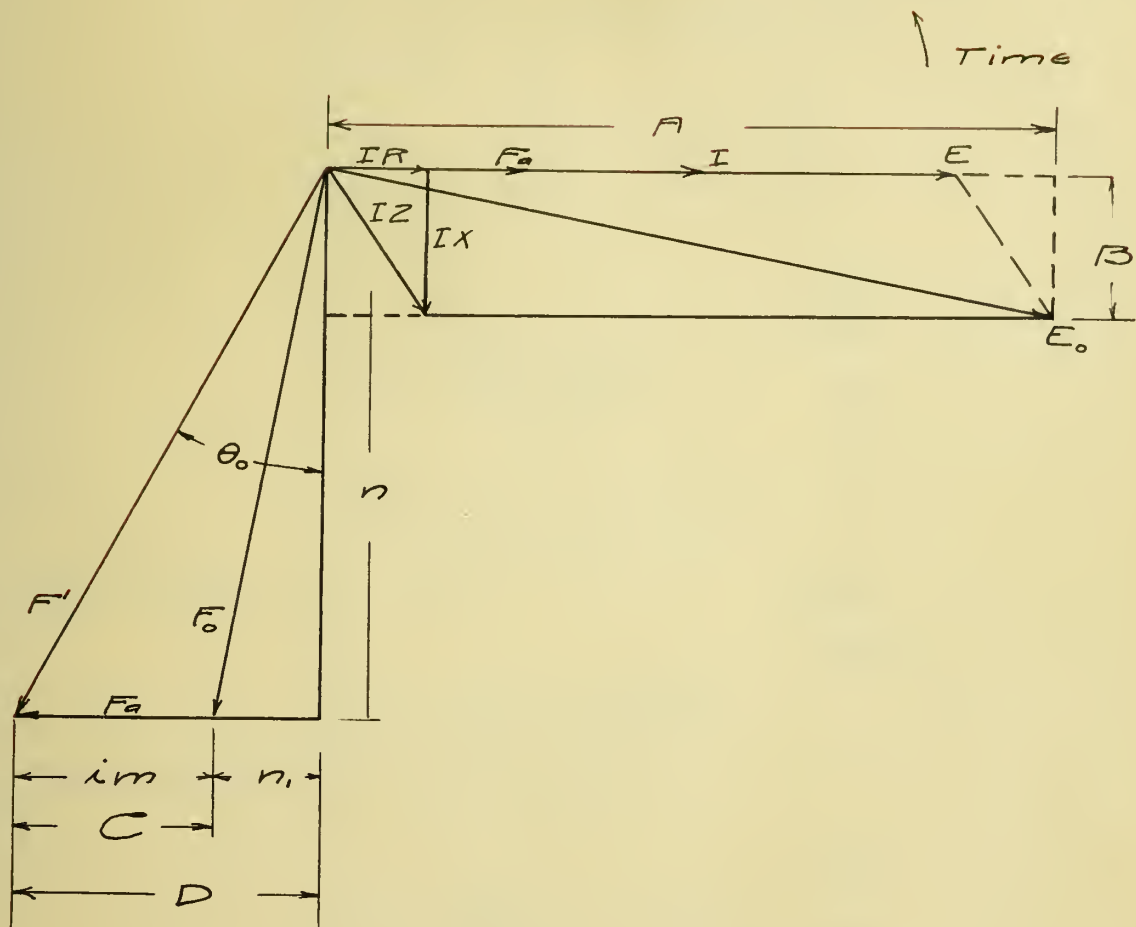
Figure 12
Vector Diagram
of
A T B - 12 - 300 - 600 - 2300 V.
Full Load
Unity Power Factor

In Fig. 12,

E = terminal E. M. F.

Terminal E   -   2300 volts

All values stated in per cent.

Terminal E %   -   1

Load   1

Therefore I   1

I R, given   .02

$A = E + I R$ 1.02

x from $\dfrac{2400 - 1910}{6600}$ .075

I x   .075

n from saturation curve

corresponding to $E + I R$ 1.025

$\dfrac{B}{A}$ .0735

n_1, by proportionality,

assuming straight line saturation, =

$\dfrac{B}{A}$ n

                                                        .075

m_1 from $\dfrac{1910}{6600}$, in m.m.f.   .29

$C = mi$ .29

$D = C + n_1$ .365

$\tan \theta_o = \dfrac{D}{n}$ .355

θ_o, electrical degrees 19.6

We have, then

$$f_H = \frac{19.8}{\sqrt{\theta_o^\circ}} = \frac{19.8}{\sqrt{19.6}} = 4.47 \text{ beats per sec.}$$

To determine the value of flux Φ it is necessary to find the area swept over by any one conductor. The total angle of swing is $2\,\theta_0 = 2 \cdot 19.6$ or 39.2 electrical degrees. Reduced to space degrees this becomes

$$\frac{39.2}{p\,/\,2} = 39.2 \cdot \frac{2}{12} = 6.54°$$

Area swept over, then, is

$$A = 1 \cdot 2\,\theta_0 \cdot \rho' \cdot \frac{1}{57.3}$$

$$= 10 \cdot 6.53 \cdot 18 \cdot \frac{1}{57.3}$$

$$= 20.5 \text{ sq. in.}$$

Total flux cut is

$$\Phi = A \cdot 60000$$

$$= 20.5 \cdot 60000 = 1,230,000 \text{ lines}$$

$$= 1.23 \cdot 10^6$$

The E. M. F. induced per bar is

$$E = 4.44\,f_H\,N\,\Phi \cdot 10^{-8}$$

Here the value of N is $1/2$ because one bar serves as one half of a complete turn.

$$E = 4.44 \cdot 4.47 \cdot \frac{1}{2} \cdot 1.23 \cdot 10^6 \cdot 10^{-8}$$

$$= .122 \text{ volts.}$$

and current per bar is

$$I = \frac{E}{r} = \frac{.122}{.0000588}$$

$$= 2075 \text{ amperes}.$$

So that loss per bar is

$$r I^2 = \overline{2075}^2 \cdot .0000588$$

$$= 253 \text{ watts}.$$

Total loss is

$$P = p \cdot N \cdot I^2 r$$

$$= 12 \cdot 6 \cdot 253$$

$$= 18.2 \text{ K.W}.$$

Approach the calculation of K_1 from the mechanical side using power distribution over 360 degrees as explained on page 21. We have then

$$K_1 \frac{d \theta}{d t} = \rho' F = \frac{HP \cdot 33000}{2 \pi \cdot R.P.M.}$$

$$= \frac{18.2}{.746} \cdot \frac{33000}{2 \pi \cdot 4.47 \cdot 60}$$

$$= 478 \text{ lb. ft}.$$

Whence

$$F = \frac{478}{1.5} \qquad = 318 \text{ lbs}.$$

the damping pull due to the amortisseur winding, at maximum hunting angle.

The angular velocity

$$\frac{d \theta}{d t} = \frac{2 \cdot \theta_0 \cdot f}{57.3} H \quad . \quad \text{radians per second}$$

$$= \frac{2 \cdot 6.53 \cdot 4.47}{57.3} .$$

$$= 1.02 \text{ radians per second.}$$

and

$$K_1 = \frac{478}{1.02} = 468$$

Now let us check this value from the expression derived on page 34.

$$K_1 = .47 \frac{p \; l^2 \; B^2}{R} \rho'^2 . \; 10^{-16}$$

$$= \frac{.47 \cdot 12 \cdot 10^2 \cdot 2.54^2 \cdot 60000^2 \cdot 1.5^2 \cdot 30.4^2}{6.45^2 \cdot .00014} . \; 10^{-16}$$

$$= 468 \qquad \text{and } K = 2940$$

where the resistance of one circuit is

$$R = 2.85 \; r'$$

r' being the resistance of a single rotor bar without the 20 per cent correction.

The next factor of interest is $\frac{K}{2 \cdot J}$, termed the damping coefficient. It comes into the equation of hunting in negative form as the coefficient of t in the exponent of ε and so is directly a measure of the damping,-- the rapidity of elimination of the hunting swing, in other words, it is the attenuation constant. It may be mentioned here that it has become customary to estimate the duration of a transient by the time which it would last if maintained at
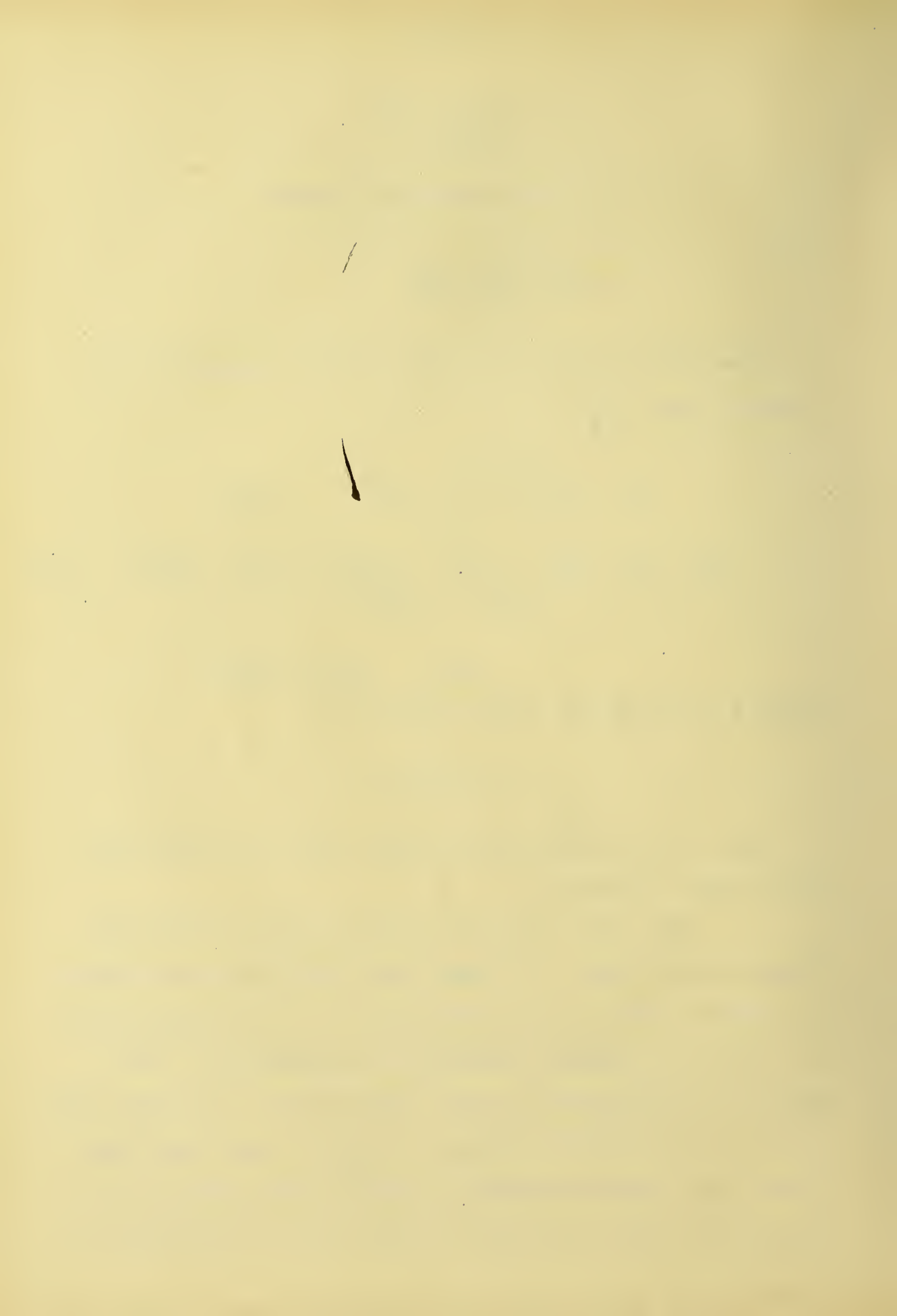
its initial value. This time value is taken as the recipro-
cal of the coefficient of t in the exponent of ϵ . In the
case calculated, the attenuation constant is

$$\alpha = \frac{K}{2 \ J} = \frac{2940}{2 \cdot 78.8} = 18.7$$

It all means that the hunting transient dies down at the rate
indicated in Fig. 13. The co-ordinates need no explanation
further than that the ordinates are in per cent of maximum
hunting which, of course, is the condition at which counting
time is begun. The curve shows that after .25 seconds hunt-
ing has, under the conditions of damping, in this particular
problem, practically ceased. This brings us to a considera-
tion of the main error due to our assumptions. As K and K_1 have
been calculated it has been assumed that oscillation con-
tinues thru maximum swing thru the duration of the transient.
It is seen from Fig. 13 that after the first instant the
damping action causes a sharp decrease in the amplitude of
the angle of oscillation. This, however, means that fewer
lines of force are cut by each conductor, less E. M. F. is
induced, less current flows, and less power is lost, so that
the damping force F diminishes. K, however holds a constant
value and is dependent only on the constants of the machine
in question.

In the following pages are tabulated and plotted
the results worked out by the method outlined for the machine
whose constants were given, for various sizes of rotor bars.
All calculations are for 6 bars per pole. As shown in Fig. 14

Figure 13
Graph
of
$y = \varepsilon^{-18.7t}$

$\varepsilon^{-18.7t}$

Time - Seconds

Calculations on Damping for Various Rotor Bar Sizes
ATB-12-300-600-2300 V.
6 Bars per Pole

Copper Size	Diam. in.	Resistance r'	r=1.2 r'	Current	P K.W.	% Total Capacity	$K\frac{d\theta}{dt}$	K_1	$\frac{f_1}{2J}$	Torque lb.
# 1	.289	.000124	.000149	818	7.2	2.4	189	185	1.17	126
0	.325	.0000983	.000118	1034	9.1	3.04	239	234	1.48	159
00	.365	.000078	.0000935	1305	11.45	3.8	301	295	1.87	201
000	.410	.0000613	.0000735	1606	14.4	4.8	378	370	2.34	252
0000	.460	.000049	.0000588	2075	18.2	6.1	478	468	2.97	319
c.m.										
250000	.500	.0000415	.0000498	2450	21.6	7.2	567	550	3.49	378
300000	.5477	.0000346	.0000415	2940	25.9	8.6	679	665	4.22	452
350000	.5916	.0000296	.0000355	3440	30.3	10.1	795	778	4.93	530
400000	.6325	.0000259	.0000311	3920	34.5	11.5	905	886	5.62	603
450000	.6708	.0000230	.0000276	4420	38.8	12.9	1018	956	6.32	678
500000	.7071	.0000208	.0000249	4900	43.1	14.3	1131	1110	7.06	780
650000	.8062	.0000160	.0000192	6350	55.8	18.6	1465	1436	9.1	977

Figure 14
Variation
of
Damping Ratio
with
Rotor Bar Diameter

Rotor Bar Diameter – Inches
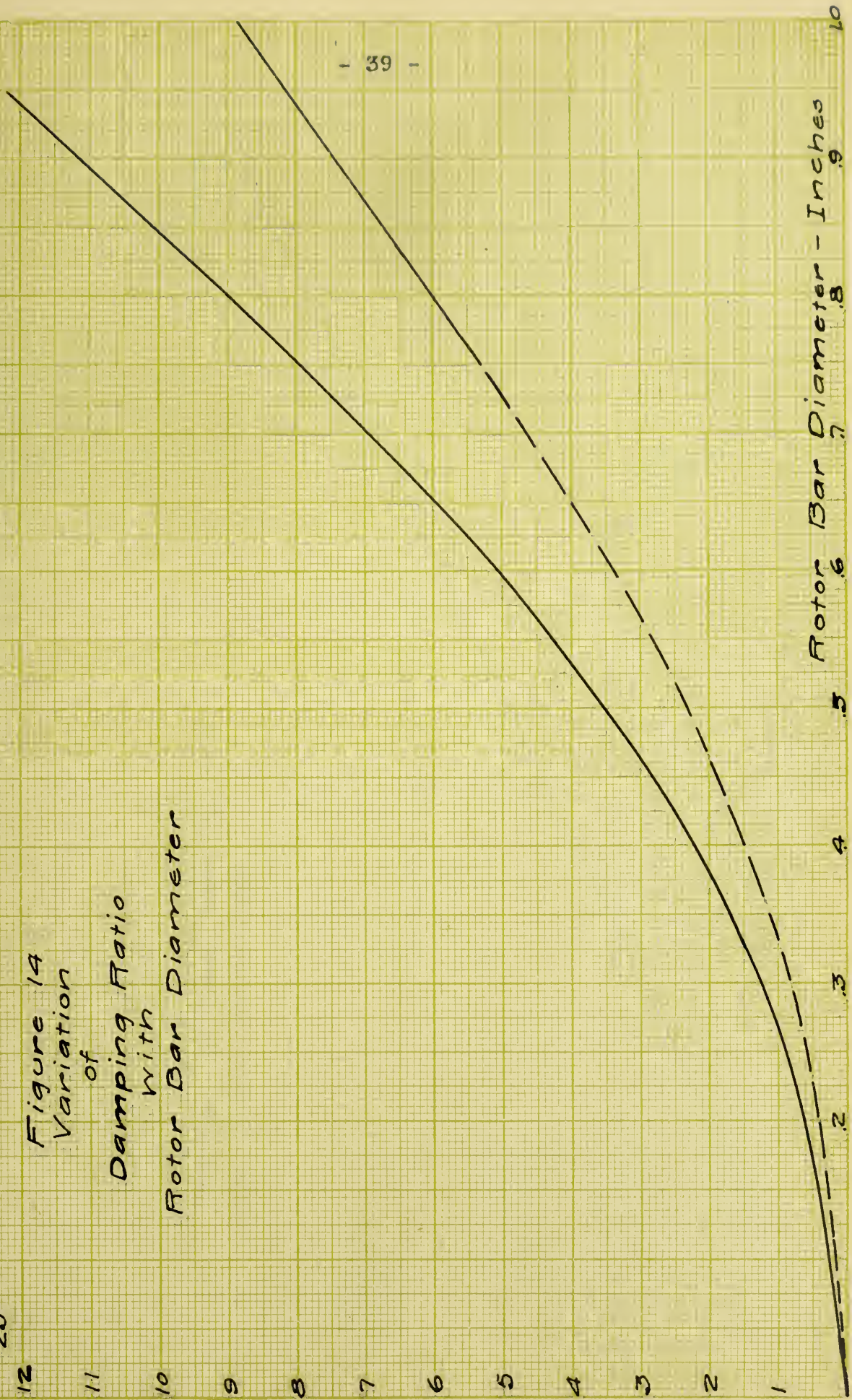
Figure 15

Variation
of
Damping Ratio
with
Per Cent of Loss
ATB-12-300-600-2300V.
6 Bars per Pole

to find the damping ratio for fewer or more bars of any
diameter it is necessary only to take proportionate ordinates
in comparison with the original curve for six bars. From
Fig. 15 it is seen that a straight line relation holds be-
tween the damping coefficient and the per cent loss in the
amortisseur. It is evident that there is a straight line
relation, also, between the damping coefficient and the
damping torque.

IV High Frequency Losses

In the calculations up to this point the losses
due to circulating eddy currents have been neglected. These
losses occur whenever the machine is in operation and must
be considered as a result of the action of and be charged
to, the damping device. The currents are caused by E. M. F.s
induced in the rotor bars by the variation in flux between
the bars caused by tufting from the stator teeth and are
therefore of very high frequency. The loss occasioned by
these eddies does not exert a damping action, but it is of
interest to calculate it in order to find what effect the
amortisseur winding has on the efficiency of the machine.
The current paths are very complicated and depend largely on
the relations between slot pitch and rotor bar pitch. The
method of handling the problem will be similar to that used
to calculate the hunting loss. The E. M. F. induced per bar

will be found and the current which flows will be determined
by use of a resistance value r = 1.2 r', allowing 20 per cent
for contact and end ring resistance. The total loss will be
the loss per bar multiplied by the number of bars in the
rotor. Due to the high frequency it will be necessary to
consider the reactance of the rotor slot.

The familiar formula

$$E = 4.44 \ f_s \ N \ \Phi \cdot 10^{-8}$$

will again be used to determine the induced E. M. F. per
bar. Here again N will be taken as $\frac{1}{2}$ of a coil. The fre-
quency f_s is measured by slot frequency and is derived as
follows:

Let S = peripheral speed -- inches per second.

l_p = slot pitch - inches

$$f_s = \frac{s}{l_p} \quad \text{or} \quad = p \ f$$

$$= 12 \cdot 60 \qquad\qquad = 720 \text{ cycles per second.}$$

It will be necessary to plot the flux distribution
and penetration into the pole to determine the value of Φ
to be used. As indicated in Fig. 16, the flux has its max-
imum value in the pole face directly opposite a tooth. Then,
due to increasing reluctance of the air gap, its value falls
off to a minimum opposite the middle point of the slot open-
ing. Assuming the maximum value as 100 per cent, the other
values for any points along the periphery will be

$$B = \frac{100}{I_t}$$

Figure 16
Flux Distribution in Pole
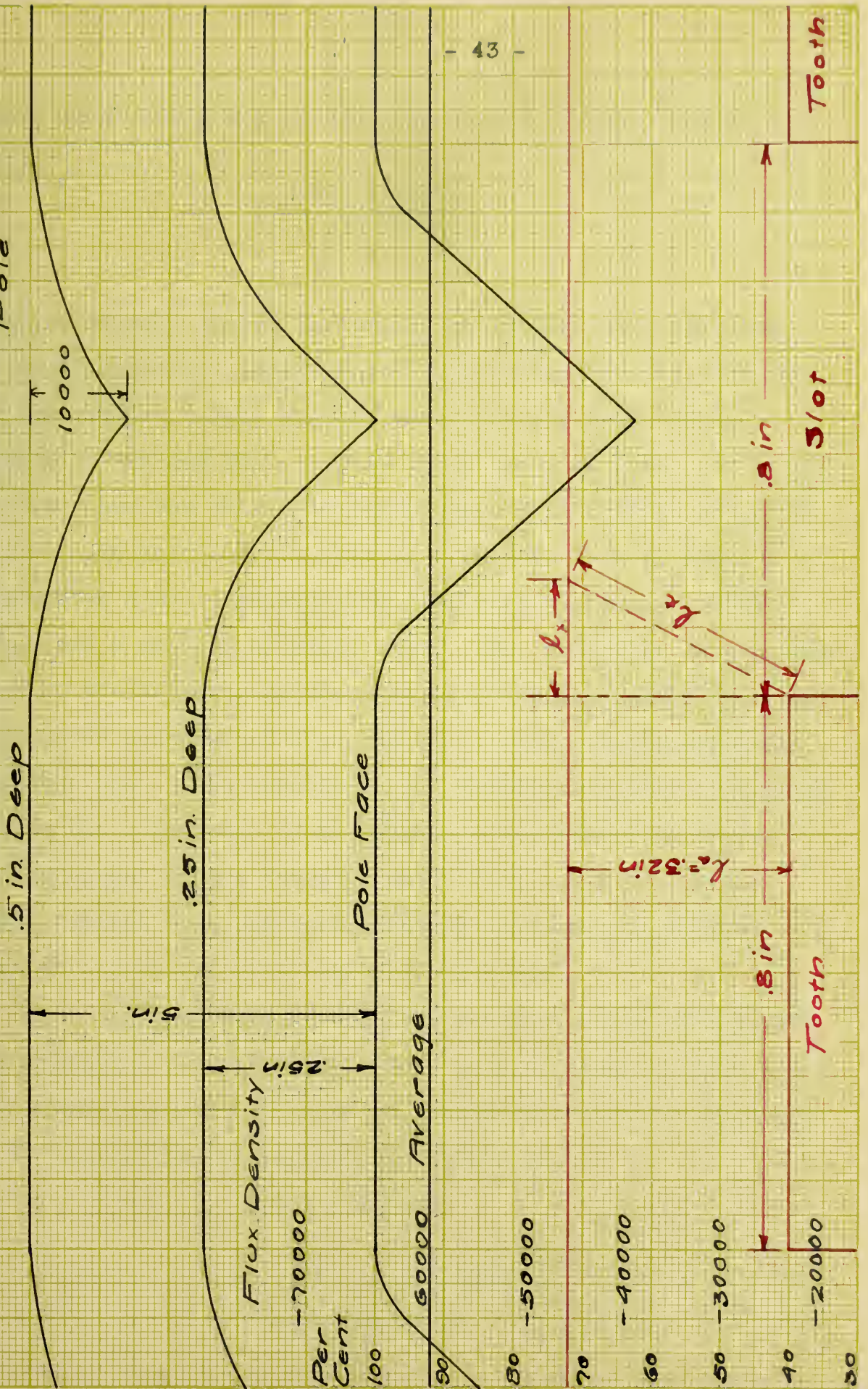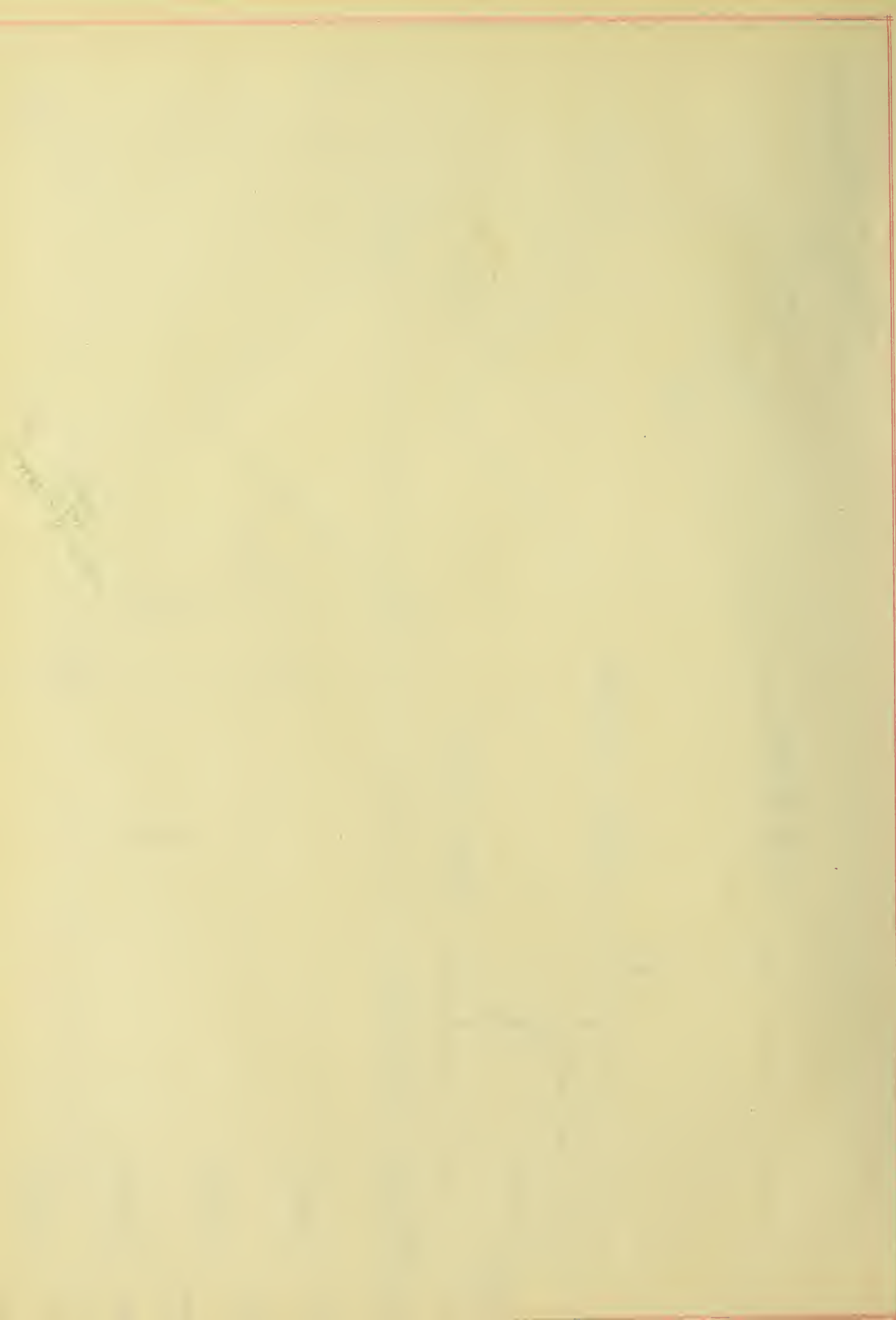
where l_t is expressed in per cent of the air gap length, so that

$$B = \frac{100 \cdot l_a}{\sqrt{l_x^2 + l_a^2}}$$

Now the average value, the one given in the specifications as 60000 lines per square inch, lies on the average ordinate of the plotted curve, and the ordinates are laid off and marked to the new scale. This is at the pole face, and it is seen that the flux pulsates between values 65710 and 38920 lines and is equivalent to a uniform flux of 60000 superposed upon an alternating flux with a maximum of 5710 lines and a minimum of 21080 lines. The pulsation of magnetic flux in the interior of the pole may be approximated by drawing curves equidistant from the one showing distribution at the pole face. The magnitude of pulsation rapidly disappears due to the reluctance of the iron and the screening or reactive effect of the currents which flow. Let us use the value derived from the curve showing distribution in the pole at a distance of $\frac{1}{2}$ inch from the face.

As the value of Φ in the equation use $\frac{1}{2}$ of the maximum pulsation at that depth. This is $\frac{1}{2}$ of 10000 or 5000 lines. The total flux cut per cycle is length of rotor bar, multiplied by slot pitch, multiplied by density or

$$\Phi = 1 \cdot l_p \cdot B$$

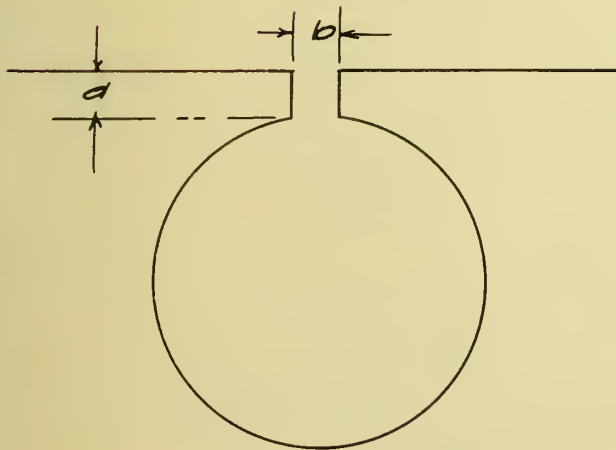$$= 10 \cdot 1.6 \cdot 5000$$

$$= 80,000 \text{ lines}$$

Since the calculation is only an approximation it is permissible, also, to use the coefficient 4.44 which is determined by the pole arc. The E. M. F. induced per bar is then

$$E = 4.44 \ f_s \ N \ \Phi \cdot 10^{-8}$$

$$= 4.44 \cdot 720 \cdot \frac{1}{2} \cdot 80000 \cdot 10^{-8}$$

$$= 1.28 \text{ volts per bar}$$

The reactance of a round slot is, Fig. 17.

$$x = 2 \ \pi \ f_s \ 1 \cdot 3.19 \ (.545 + \frac{d}{b}) \ a^2 \cdot 10^{-8}$$

where

1 is effective
length of conductor-inches
f_s is frequency,-
slot in this case
d = b is assumed
a is effective
number of conductors per
slot - one in this case.

Figure 17

Substituting values

$$x = 2 \ \pi \cdot 720 \cdot 10 \cdot 3.19 \ (.545 + 1) \ 1^2 \cdot 10^{-8}$$

$$= 2 \ \pi \cdot 720 \cdot 10 \cdot 3.19 \cdot 1.545 \cdot 10^{-8}$$

$$= .00223 \text{ ohms}.$$

On account of the high frequency it will be necessary to consider skin effect. From Steinmetz' Transient Phenomena

the effective resistance is increased from the ohmic resistance r to the value

$$r_o = C \, l_o \, r$$

where

$$C = \sqrt{.4 \, \pi^2 \cdot \lambda \cdot \mu \cdot f \cdot 10^{-8}}$$

λ is electric conductivity of material.

f is frequency of impressed E. M. F. - f_s.

μ is magnetic permeability of conductor material

l_o is radius of conductor, - cm.

From Steinmetz' tables

$$\lambda = 6.2 \cdot 10^5$$

$$\mu = 1$$

so that

$$C = (.4 \, \pi^2 \cdot 1 \cdot 6.2 \cdot 720 \cdot 10^5 \cdot 10^{-8})^{\frac{1}{2}}$$

$$= 4.19$$

and

$$r_o = C \, l_o \, r$$

for the #0000 bars

$$= 4.19 \cdot \frac{.460}{2} \cdot 2.54 \cdot .0000588$$

$$= .000144 \text{ ohms effective.}$$

Consider the worst possible case of conductor spacing that is at $\frac{1}{2}$ slot pitch. The current which flows will be

$$I = \frac{E}{Z} = \frac{E}{\sqrt{r^2 + x^2}}$$

$$= \frac{1.28}{\sqrt{.000144^2 + .00223^2}}$$

$$= 573 \text{ amperes.}$$

Power per bar

$$P' = I^2 r_\rho$$
$$= 573^2 \cdot .000144$$
$$= 47.2 \text{ watts}$$

Total loss

$$P = p \, N \, P'$$
$$= 12 \cdot 6 \cdot 47.2$$
$$= 3.4 \text{ kilowatts}$$

or 1.133% of machine capacity.

In the following pages are tabulated and plotted the results, worked out by the method just indicated, of eddy current values for amortisseur windings of various sizes and therefore various damping ratios, on the machine under consideration thruout this paper. This high frequency loss depends entirely on the pitch of the rotor bar winding; it is greatest when rotor bar pitch is equal to one half of machine slot pitch, and disappears entirely when bar pitch and slot pitch are equal. The curve in Fig. 20 shows the total loss, the combined loss of hunting and eddies, for various damping ratios for the machine used, with 6 rotor bars per pole.

V   Starting

One of the advantages of an amortisseur winding in a synchronous motor is the fact that it makes starting

High Frequency Eddy Loss for Various Rotor Bar Sizes
ATB-12-300-600-2300V.
6 Bars per Pole.

Wire Size	Diam. in.	Resistance $r=1.2r'$	Resistance $6=r/6C$	P K.W.	% Total Capacity	$\frac{Kl}{2J}$	Total Loss %
#							
1	.289	.000149	.000225	5.41	1.81	1.17	4.20
0	.325	.000118	.000204	4.82	1.61	1.48	4.65
00	.365	.0000935	.000182	4.3	1.43	1.87	5.23
000	.410	.0000735	.000160	3.78	1.26	2.34	6.06
0000	.460	.0000588	.000144	3.4	1.13	2.97	7.23
c.m.							
250000	.500	.0000498	.000132	3.16	1.05	3.49	8.25
300000	.5477	.0000415	.000121	2.86	.95	4.22	9.55
350000	.5916	.0000355	.000112	2.65	.88	4.93	10.98
400000	.6325	.0000311	.000105	2.48	.83	5.62	12.33
450000	.6708	.0000276	.0000985	2.33	.78	6.32	13.68
500000	.7071	.0000249	.0000963	2.21	.74	7.06	15.04
650000	.8062	.0000192	.0000823	1.95	.65	9.10	19.25

Figure 18
Variation
of
Eddy Loss in Rotor Winding
With
Rotor Bar Diameter
ATB-12-300-600-2300 V.
6 Bars per Pole
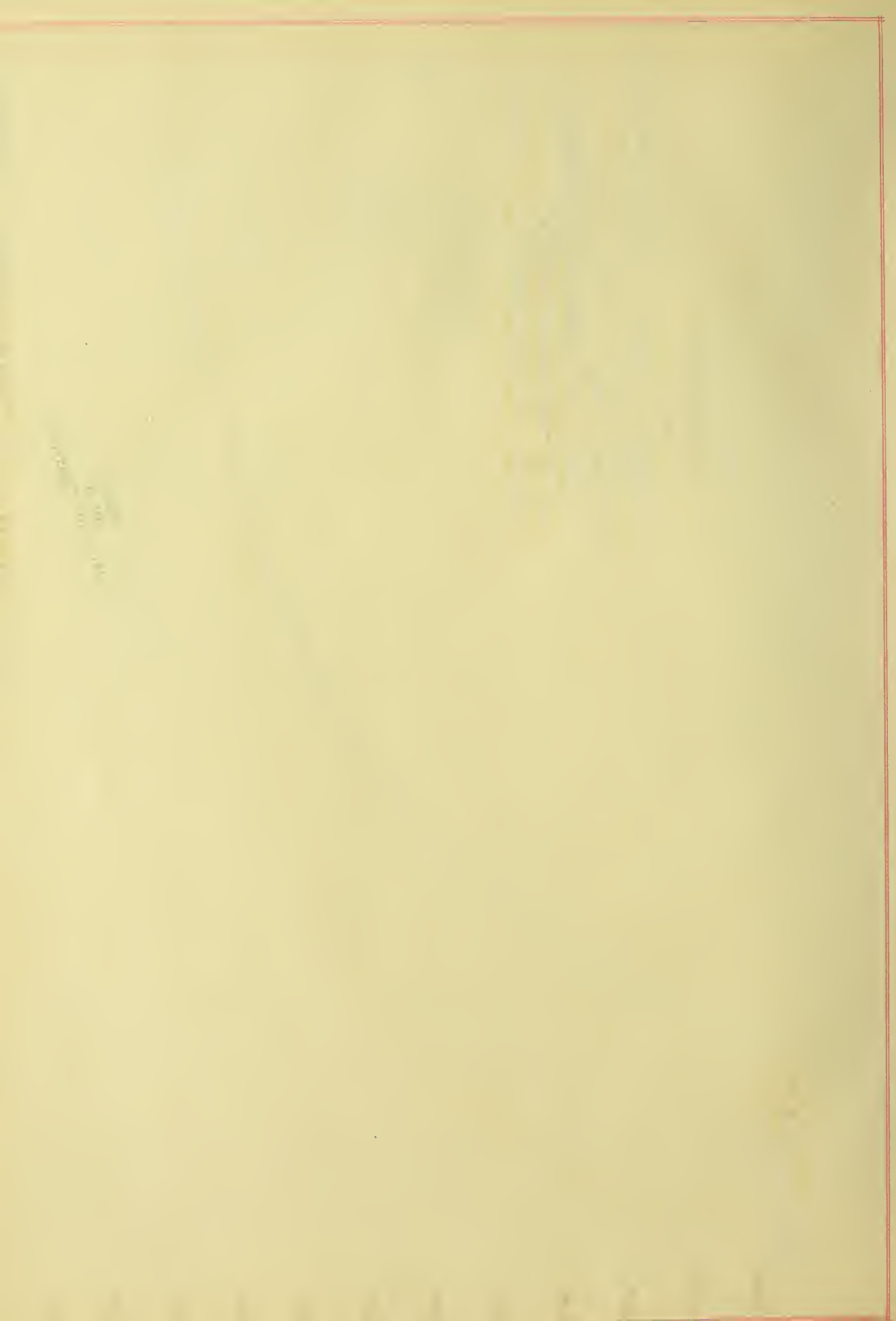Slot Frequency ~ 720~

Diameter of Rotor Bars ~ Inches

Per Cent Loss

Figure 19
Variation
of
Per Cent Loss — Eddy
with
Damping Ratio
ATB-12-300-600-2300V
6 Bars per Pole
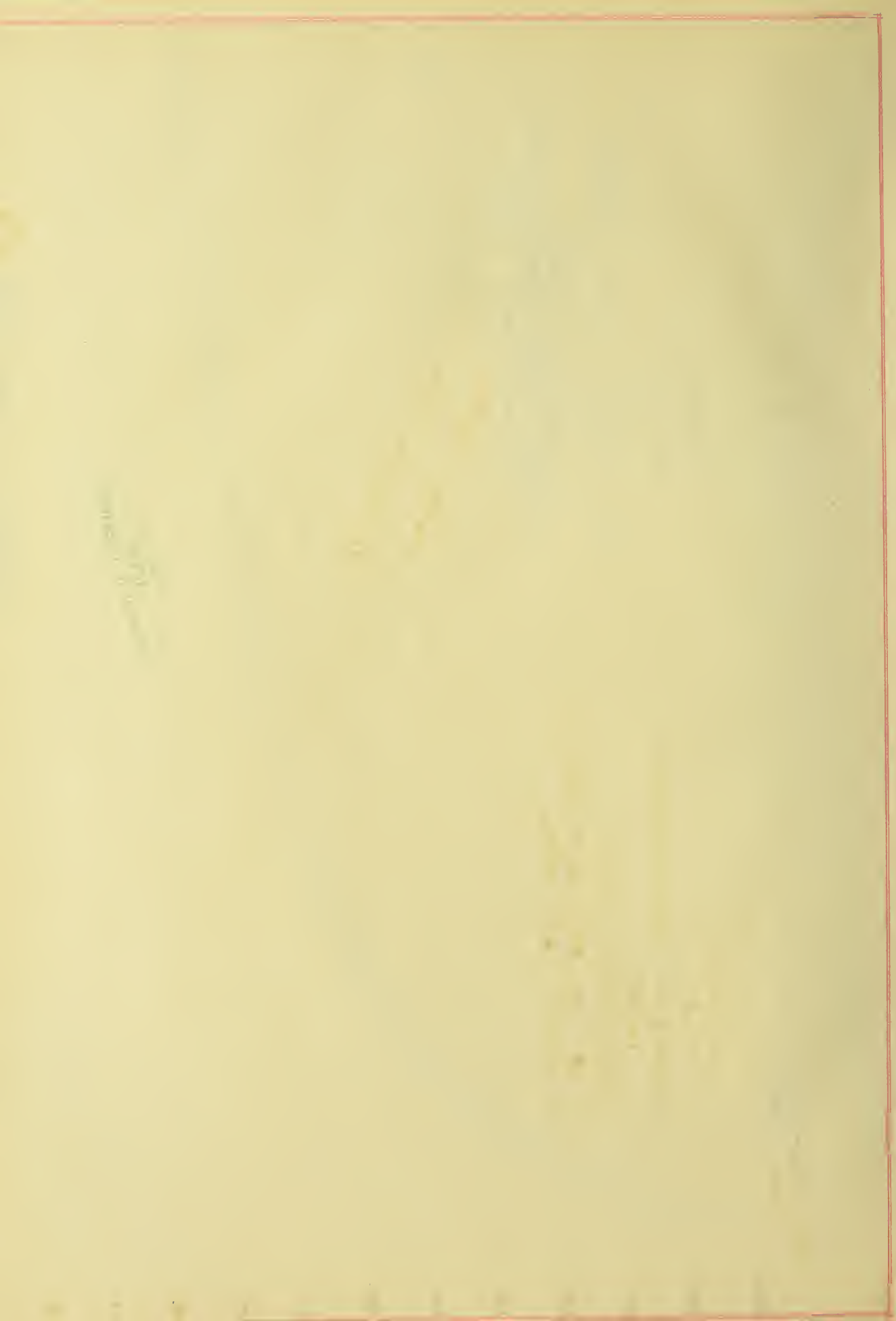
Per Cent
Loss

Damping Ratio $\frac{\tau}{2t}$

Per Cent
Loss

Figure 20
Variation
of
Losses in Amortisseur
with
Damping Ratio
ATB-12-300-600-2300V.
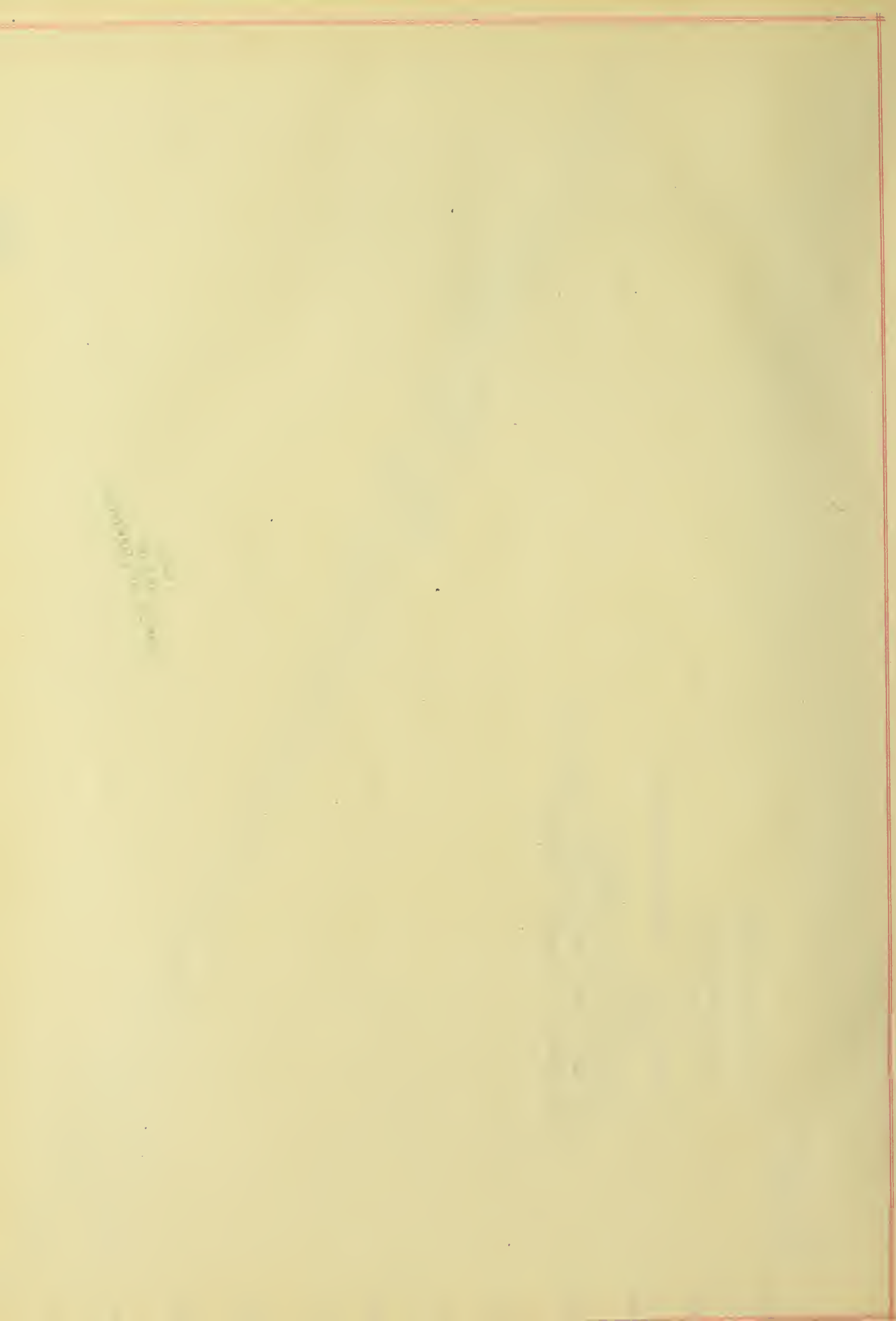6 Bars per Pole

Total Loss Alone

Hunting Loss

$\dfrac{K_1}{2J}$

very much easier. The amortisseur acts exactly as an in-
duction motor rotor. The starting torque is furnished by,
and is proportional to, the components of the currents in
the rotor circuit which are in phase with the electro-motive
forces that produce them. The question of rotor bar pitch
in comparison with armature slot pitch again comes up in this
connection. Due to varying flux path reluctances there is a
non-uniformity of torque at the instant of starting from
various positions. When the rotor is in a position of minimum
reluctance the magnetic forces will tend to hold it there and
cause "magnetic locking". The worst condition for this effect
is when rotor bar pitch and slot pitch are equal. Therefore
this arrangement is to be avoided even if the high frequency
loss in the rotor winding disappears. The best condition for
starting is when bar pitch is one half of slot pitch, in which
case the high frequency loss is greatest. As a compromise,
therefore it may be suggested that rotor bar slots be arrang-
ed with a pitch equal to three fourths of machine slot pitch.
Of course, corresponding multiples of pitch arrangement pro-
duce similar results.

VI Heating

It would seem, from the calculations carried thru,
that the currents which flow in the amortisseur windings must
be excessive. They are, indeed, from the usual design stand-
point of amperes per square inch comparisons. It must be

remembered that these currents flow for very short periods
of time only, that the radiating conditions for the bare wind-
ing, partly imbedded in iron pole faces, are particularly
favorable, and that the temperature is limited only by the
extent to which the surrounding coils will be heated.

VII Conclusion

        In conclusion, it may be well to restate a defini-
tion of hunting and the damping curves for it. Hunting is
a pendulum action by the rotating mass of a machine gone thru
in coming back to its proper position after disturbance of
its uniform rotation. The change in position causes energy
currents to flow which accelerate the lagging and retard the
leading machine. Their magnetic action is a distortion or
shift of field. The resultant magnetic attraction opposes
their separation and pulls them together. This, with fric-
     as a damper
tion, tends to keep the two machines in step. The frequency
of hunting depends on the magnitude of this magnetic attrac-
tion, that is on the field excitation, and on the weight and
moment of inertia of the rotating parts. The hunting may be
stopped by increasing the energy losses due to the oscilla-
tion by means of copper bridges between the poles, by metal
collars around the pole faces or by a complete squirrel cage
winding in the pole faces.

HUNTING OF SYNCHRONOUS MACHINES

VIII Index - Notation

a - Twisting moment per unit angular displacement

A - Maximum area swept over in half of one complete vibration.

A. T. - Ampere turns

α - Damping ratio = $\frac{K}{2} \frac{}{J}$

B - Flux density

e - Instantaneous E. M. F. induced per bar.

e_o - Impressed E. M. F. - Generator

e_c - Counter E. M. F. - Motor.

E - Effective E. M. F. induced per bar.

f - Frequency of machine - cycles per second

f_H - Frequency of hunting - beats per second

f_s - Slot frequency - cycles per second

F - Force acting on the periphery of the rotor

F_o - Ampere turns to generate e_c

F' - Ampere turns actually required

H.P. - Horse power.

I - Effective current per bar

J - Moment of inertia of rotor.

K - Moment of retarding force per unit angular velocity

K.W. - Kilowatts

l - Effective length of rotor bar.

l' - Total length of rotor bar

L - Disturbing moment

l_p - Slot pitch

M - Mass of rotor

N - Number of rotor bars per pole

p - Number of poles

P - Total power loss in amortisseur

P' - Power loss per circuit in amortisseur

Q - Constant $= \dfrac{K.W.}{J}$

r - Effective or corrected resistance per rotor

bar $= 1.2$ r'

r' - Actual resistance per rotor bar

R - Resistance per circuit of amortisseur

R. P. M. - Revolutions per minute.

ρ - Radius of Gyration

ρ' - Radius of rotor, ϖ of amortisseur winding

S - Peripheral speed of rotor

T_1 - Torque

θ - Angular displacement in hunting

θ_1 - Some particular form of θ.

θ_0 - Maximum angle of swing in hunting.
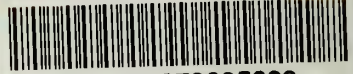
T - Time of one complete period.

x - Rotor slot reactance.

K_1 - Moment of retarding force per unit angular velocity translated from pendulum oscillation to uniform rotation.